Bridging work for Maths in Context

Maths department

2022/2023

Name___________________

Holmer Green
SENIOR SCHOOL

Maths in Context – Bridging Work
Holmer Green Senior School

In order to achieve in Level 3 Certificated Maths in Context it is **vital** that you have a secure knowledge of GCSE Mathematics content. In particular, you must be **fluent** in the following topics:

- Sets and Venn Diagrams.
- Quadratic sequences.
- Probability and Venn Diagrams.
- Exponential growth and decay.
- AER and compound interest.
- Using spreadsheets: organising data.
- Linear programming.
- Gradient of graphs.
- Times series graphs.
- Risks.

We expect that most students will already be confident in the vast majority of these topics.

It is essential that all students spend a significant amount of time practising these topics at regular intervals between the end of Year 11 and the start of Year 12.

Mathematical fluency does not simply mean that you have met this topic before and think that you remember how to do it. To reach fluency, you must be able to **quickly** and **accurately** recall concepts and methods.

Mark your work using the 'Answers' attached at the end of every topic, checking that you have understood.
If you find that you have made mistakes, **identify** and **correct** these. If you cannot do this, reread the 'Examples' for that specific topic, to ensure that you have not misunderstood a concept. If you still do not understand something and cannot understand why, you are welcome to email the Head of Maths, Mr Ortega, at ortegaj@holmer.org.uk for further resources.
Complete and mark the 'Extend' questions to make sure that you do have an excellent understanding.

Please bring all of your **completed and marked** bridging work to your first maths lesson where it will be checked by your maths teacher. We expect you to complete the questions on lined or squared paper, showing a **full method** and **working out**.

There will be a baseline assessment covering these topics in the first weeks of Year 12. It is expected that all Maths in context students will demonstrate an excellent understanding of all topics in this assessment.

M1.1 Sets and Venn diagrams

Before you start

You should be able to:
- identify numbers that have common properties.

Why do this?

It is useful to be able to classify objects by their characteristics. Scientists frequently classify animals and plants using their different characteristics.

Objectives

- You can use Venn diagrams to represent sets.
- You can interpret Venn diagrams.
- You can draw a Venn diagram using given information.

Get Ready

How would you describe these numbers?

1 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
2 5, 10, 15, 20
3 2, 3, 5, 7, 11, 13

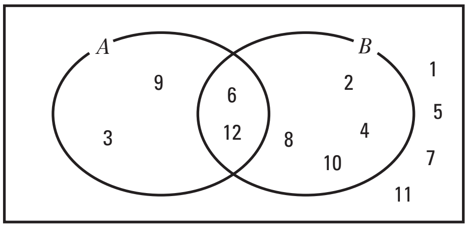
Key Points

- A set is a collection of numbers or objects. For example, if W is the set of the first ten whole numbers then this can be written as:

  $W = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

  The whole numbers from 1 to 10 are the members of set W.
- A picture called a Venn diagram is used to represent sets and show the relationship between them. For example, the following Venn diagram shows:

  all the whole numbers from 1 to 12
  the set A where $A = \{3, 6, 9, 12\}$
  the set B where $B = \{2, 4, 6, 8, 10, 12\}$



  All the members of set A are inside the circle labelled A.
  All the members of set B are inside the circle labelled B.
  The numbers that are in both set A and set B are in the intersection of the two sets.
  The numbers 1, 5, 7, 11 are not in set A or set B so are outside the two circles.
- Venn diagrams can also be used to show the number of members in a set.

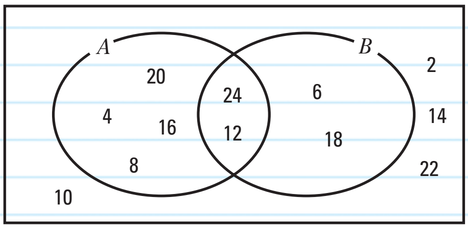
Example 1

The Venn diagram shows the even numbers from 2 to 24.

A01

Write down the numbers that are:

a in set A
b in set B
c in both set A and set B
d not in set A or set B.



a $A = \{4, 8, 12, 16, 20, 24\}$ ← Write down all the numbers inside the circle labelled A

b $B = \{6, 12, 18, 24\}$

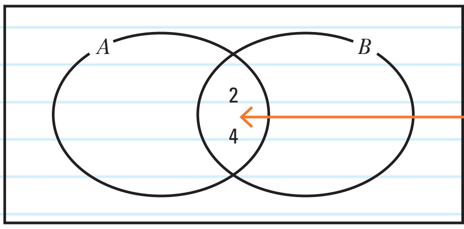
c 12, 24 ← Write down the numbers that are in the intersection of the two circles

d 2, 10, 14, 22 ← Write down all the numbers that are outside both circles

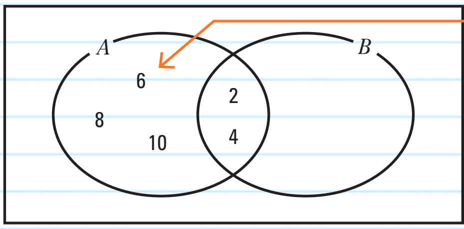
Example 2 On a Venn diagram show:

A03

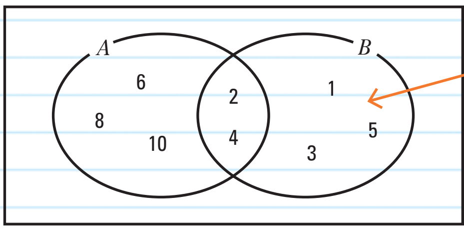
the whole numbers from 1 to 10
set A where $A = \{2, 4, 6, 8, 10\}$
set B where $B = \{1, 2, 3, 4, 5\}$



2 and 4 are in both set A and set B. Place these numbers in the intersection

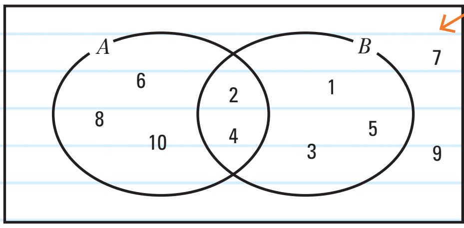


The numbers remaining from set A are 6, 8 and 10, place these in the part of circle A that does not intersect with B



The numbers remaining from set B are 1, 3 and 5, place these in the part of circle B that does not intersect with A

The whole numbers remaining from 1 to 10 are 7 and 9, place these outside the two circles.



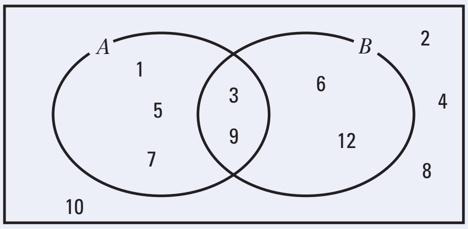
ResultsPlus
Examiner's Tip

When you have finished, check that all the numbers from 1 to 10 are somewhere in your Venn diagram

Exercise 1A

AO1 F

1

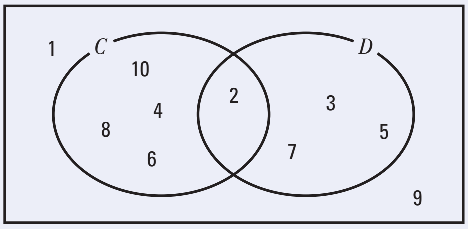


Write down the numbers that are in:

a set A **b** set B **c** both set A and set B **d** not in set A.

AO1

2



Write down the numbers that are in:

a set D **b** both set C and set D **c** not in set C.

AO3 E

3 On a Venn diagram show:

the whole numbers from 1 to 10

set A where $A = \{1, 2, 5, 10\}$

set B where $B = \{2, 4, 6, 8, 10\}$

AO3

4 On a Venn diagram show:

the whole numbers from 15 to 21

set P where $P = \{15, 18, 21\}$

set Q where $Q = \{16, 18, 20\}$

AO3 D

5 Here are some letters.

C F G H I N S T X

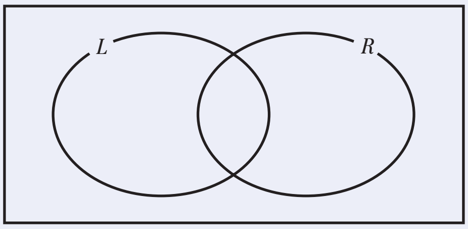
Some of the letters have line symmetry.

Some of the letters have rotational symmetry of order 2.

$L = \{\text{letters with line symmetry}\}$

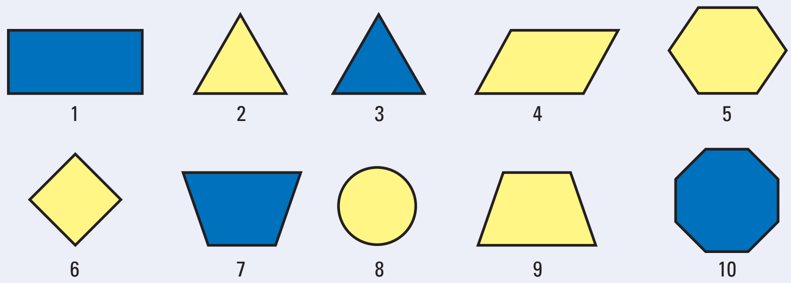
$R = \{\text{letters with rotational symmetry of order 2}\}$

Copy and complete the Venn diagram.



D A03

6 Here are some coloured shapes.

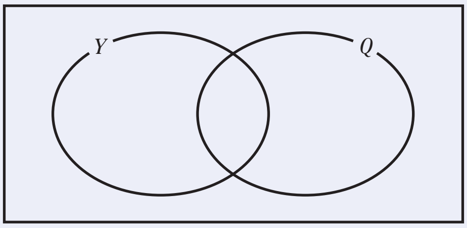


Some of the shapes are quadrilaterals.
Some of these shapes are yellow.

Y = {yellow shapes}

Q = {quadrilaterals}

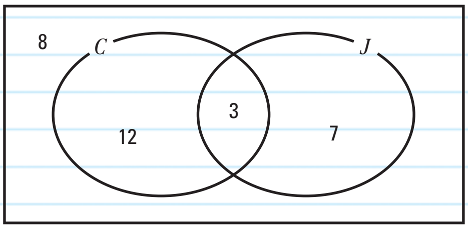
Copy and complete the Venn diagram.



Example 3

Some adults were asked whether they like classical music or jazz.

The Venn diagram shows information about their answers.



a How many adults were asked whether they like classical music or jazz?

b How many adults like jazz?

c How many adults like both jazz and classical music?

d How many adults like neither jazz nor classical music?

a $8 + 12 + 3 + 7 = 30$ ← Add up all the numbers in the Venn diagram.

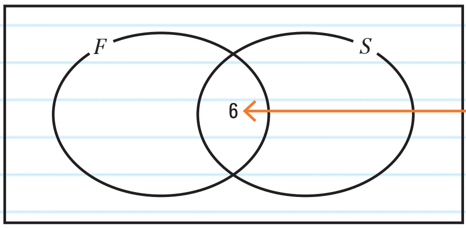
b $7 + 3 = 10$ ← Add together the two numbers inside circle J.

c 3 ← Write down the number in the intersection of the circles.

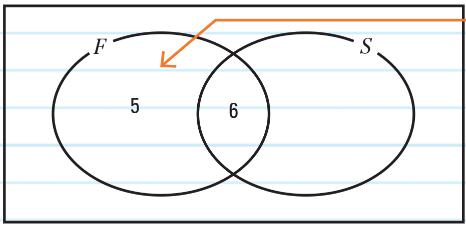
d 8 ← Write down the number outside the two circles.

Example 4

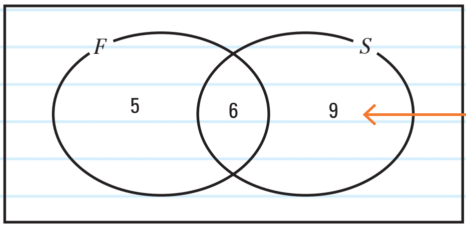
There are 27 students in a class.
11 of the students study French.
15 of the students study Spanish.
6 of the students study both French and Spanish.
Draw a Venn diagram to show this information.



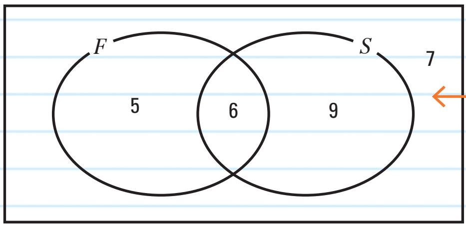
Start with the intersection. Place a 6 in the intersection to represent the 6 students that study both French and Spanish.



11 of the students study French.
$11 - 6 = 5$



15 of the students study Spanish.
$15 - 6 = 9$



$5 + 6 + 9 = 20$
$27 - 20 = 7$
So there are 7 students who do not study French or Spanish.
Place the 7 outside both circles.

Exercise 1B

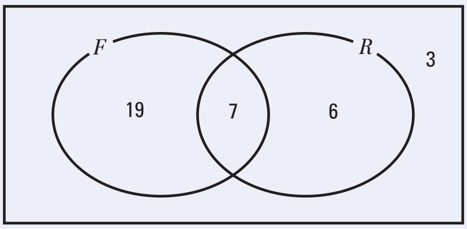
1 Some boys were asked if they played football or rugby.
The Venn diagram shows information.

a How many boys were asked if they played football or rugby?

b How many boys played just rugby?

c How many boys do not play football?

d How many boys play both rugby and football?



A03 E

D A03

2 In a class of 34 students

19 drink tea,

4 drink coffee,

3 drink both coffee and tea.

a Draw a Venn diagram to show this information.

b How many students do not drink coffee or tea?

A03

3 23 people work in a small factory.

12 are female,

10 wear glasses,

3 are female and wear glasses.

a Draw a Venn diagram to show this information.

b How many men wear glasses?

A03

4 There are 24 flowers in a bunch.

15 of the flowers are tulips,

8 of the flowers are pink,

5 of the flowers are pink tulips.

Draw a Venn diagram to show this information.

C A03

5 In a class of 31 students

15 of the students study History,

12 of the students study Geography,

7 study both History and Geography.

How many students study neither History nor Geography?

M1.2 Set language and notation

Before you start

You sholud be able to:

- find factors and multiples
- identify prime numbers.

Why do this?

In mathematics we use symbols to represent different operations. This is also true when working with sets.

Objectives

- You can use a Venn diagram to solve a problem.
- You can understand and be able to find the intersection and union of sets.

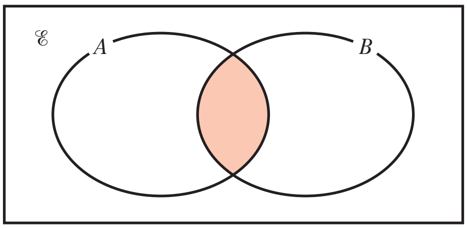
Get Ready

$A = \{1, 2, 3, 4, 5, 6\}$
$B = \{2, 4, 6, 8, 10\}$
$C = \{1, 3, 5, 7, 9\}$

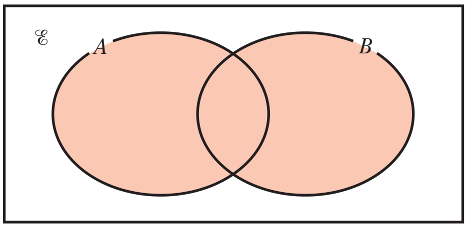
1. Write down the numbers in both A and B.
2. Write down the numbers in both A and C.
3. Write down the numbers in both B and C.

Key Points

- The **universal set** is the set of elements from which members of all other sets are selected. The symbol $\mathscr{E}$ is used to represent the **universal set**.
- A' is called the **complement** of set A. A' contains all the members of $\mathscr{E}$ that are not in set A.
- The symbol $\varnothing$ is used to represent the **empty set.** $\varnothing = \{\}$
- The symbol $\cap$ is used to represent the **intersection** of two sets. $A \cap B$ is the set of members of $\mathscr{E}$ that are in both set A and set B.



- The symbol $\cup$ is used to represent the **union** of two sets. $A \cup B$ is the set of members of $\mathscr{E}$ that are in set A or in set B or in both sets.



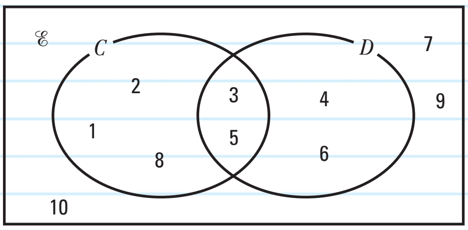
- Venn diagrams can be used to solve problems.

Example 1

The Venn diagram shows sets $\mathscr{E}$, C and D.

Write down the members of:

a the universal set, $\mathscr{E}$
b set D'
c $C \cap D$
d $C \cup D$



a $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ← List all the numbers in the Venn diagram.

b $D' = \{1, 2, 7, 8, 9, 10\}$ ← List all the numbers that are not in circle D.

c $C \cap D = \{3, 5\}$ ← List all the numbers that are in the intersection of C and D.

d $C \cup D = \{1, 2, 3, 4, 5, 6, 8\}$ ← List all the numbers that are in circle C or in circle D or in both circles.

Example 6

$A = \{2, 2, 3, 5\}$

$B = \{2, 2, 2, 3, 3\}$

The numbers in set A are the prime factors of 60.

The numbers in set B are the prime factors of 72.

a Draw a Venn diagram to show set A and set B.

b Use your Venn diagram to find:

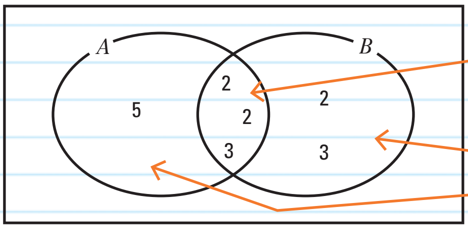
i the HCF of 60 and 72

ii the LCM of 60 and 72.

ResultsPlus Hint

You learnt how to find the prime factors of a number earlier in the course. [Mathematics A Linear Foundation chapter 1.10]

a



The two sets have the numbers 2, 2, 3 in common, put these numbers in the intersection.

Next put the remaining numbers from set A and set B on the diagram.

b **i** $A \cap B = \{2, 2, 3\}$ ← To find the HCF, find the numbers in the intersection of A and B.

$2 \times 2 \times 3 = 12$ ← Multiply these numbers together to find the HCF.

The HCF of 60 and 72 is 12.

ii $A \cup B = \{2, 2, 2, 3, 3, 5\}$ ← To find the LCM, find the numbers that make up the union of A and B.

$2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$ ← Multiply these numbers together to find the LCM.

The LCM of 60 and 72 is 360.

Example 7

a Use a Venn diagram to show:

$\mathscr{E} = \{\text{integers from 10 to 25}\}$

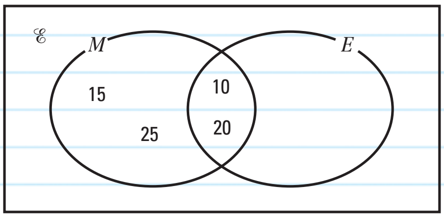
$M = \{\text{multiples of 5}\}$

$E = \{\text{even numbers}\}$

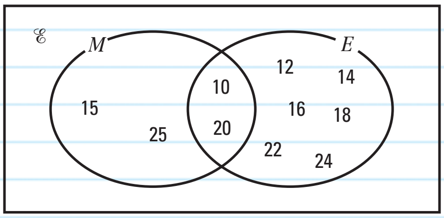
b Write down the members of:

i $M \cap E'$ **ii** $(M \cap E)'$

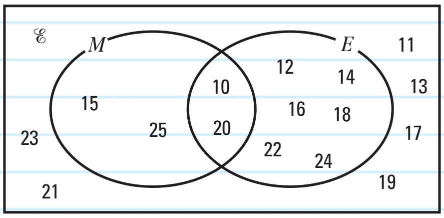
a



Place all the multiples of 5 within circle M. 10 and 20 are even numbers as well as multiples of 5 so these go in the intersection.



Place all the other even numbers within circle E.



Place all the other members of $\mathscr{E}$ that are not in set M or in set E outside the circles.

b **i** $M \cap E' = \{15, 25\}$

List the numbers that are both in circle M but not circle E.

ii $(M \cap E)' = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25\}$

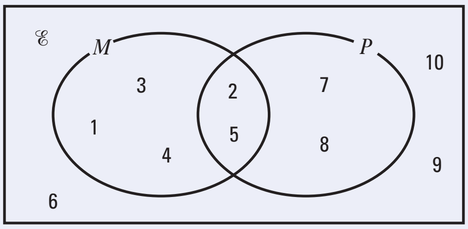
List the numbers that are not in $M \cap E$.

Exercise 1C

C

A01

1

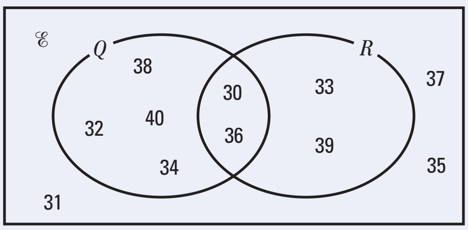


Write down the members of sets:

a P **b** $P \cap M$ **c** $P \cup M$

A01

2



Write down the members of:

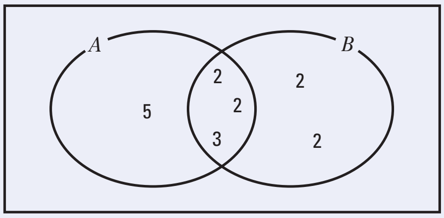
a the universal set, $\mathscr{E}$ **b** set Q **c** set R'

d $Q \cap R$ **e** $Q \cup R$

C

A03

3 The numbers in set A are the prime factors of 60.
The numbers in set B are the prime factors of 48.



Use the Venn diagram to find:
- **a** the HCF of 48 and 60
- **b** the LCM of 48 and 60.

A03

4 $A = \{\text{prime factors of } 75\}$
$B = \{\text{prime factors of } 90\}$
- **a** Write 75 as the product of its prime factors.
- **b** Write 90 as the product of its prime factors.
- **c** Show set A and set B on a Venn diagram.
- **d** Use your Venn diagram to find:
  - **i** the HCF of 75 and 90
  - **ii** the LCM of 75 and 90.

B

A03

5 **a** Draw a Venn diagram to show:
$\mathscr{E} = \{\text{integers from 12 to 24}\}$
$F = \{\text{multiples of } 4\}$
$G = \{\text{multiples of } 3\}$
b Write down the members of:
i $F \cap G$ **ii** G' **iii** $(F \cup G)'$

A03

6 **a** Draw a Venn diagram to show:
$\mathscr{E} = \{\text{integers from 20 to 29}\}$
$P = \{\text{prime numbers}\}$
$M = \{\text{multiples of } 4\}$
b Write down the members of:
i $P \cap M$ **ii** $P' \cap M$

A03

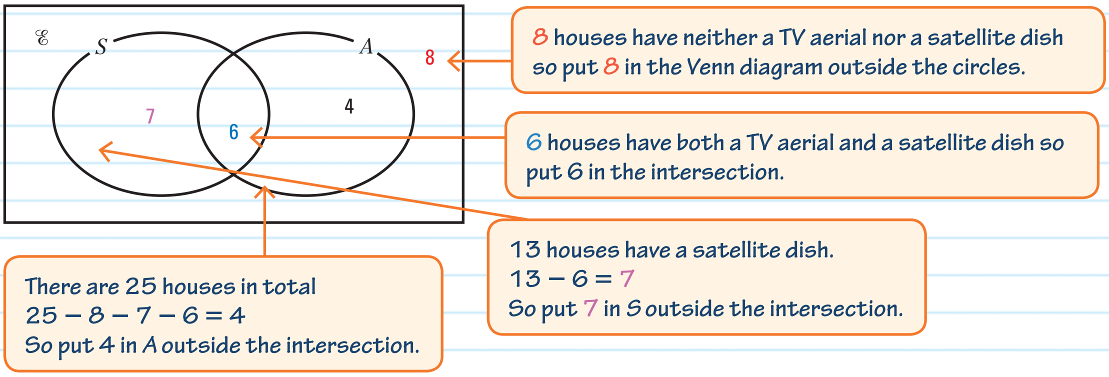
7 $\mathscr{E} = \{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$
$A = \{5, 6, 8, 9, 12, 14, 15\}$
$B = \{8, 9, 10, 11, 12, 13, 14\}$
Write down the members of:
a A' **b** $A \cap B$ **c** $A \cup B$ **d** $A' \cap B$

A03

8 $\mathscr{E} = \{\text{integers less than } 20\}$
$A = \{\text{multiples of } 5\}$
$B = \{\text{multiples of } 3\}$
a Write down the members of:
i B' **ii** $A \cup B$ **iii** $A' \cap B$
b Describe the members of $A \cap B$.

Example 8 There are 25 houses in a street.

8 of these houses have neither a TV aerial nor a satellite dish.
13 houses have a satellite dish.
6 houses have both a satellite dish and a TV aerial.
How many houses have an aerial?
One way to solve this problem is to put all the information into a Venn diagram.



$4 + 6 = 10$
There are 10 houses that have a TV aerial.

Exercise 1D

1 35 adults were asked which of two newspapers they read. A03 B

13 read the Daily Express,
28 read the Daily Telegraph,
4 read neither paper.

a Show this information on a Venn diagram.
b How many adults read both newspapers?

2 There are 60 people at an activity centre. A03

31 go sailing,
24 go sailing and go on the climbing wall,
12 do neither activity.

How many just go on the climbing wall but do not go sailing?

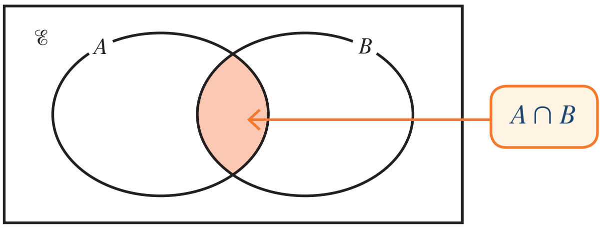
3 In a class of 31 students 18 study History, 8 study French and 5 students in the class study both History and French. A03 A
How many students study neither subject?

4 In a class 24 students play hockey, 13 play netball and 8 play both hockey and netball. How many students are there in the class if each student plays at least one of hockey or netball? A03 B

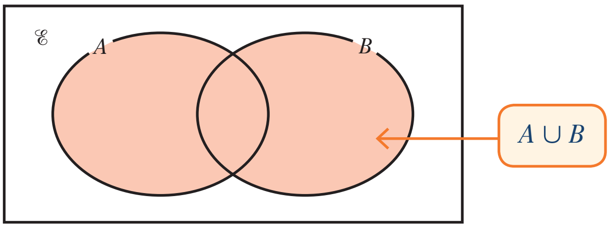
5 There are 54 customers in an Italian restaurant. There are 9 people who have finished eating. The others are eating pasta or salad or both. There are 34 eating pasta and 15 eating salad. A03
How many people are eating just pasta?

Review

- A set is a collection of numbers or objects.
- A picture called a Venn diagram is used to represent sets and show the relationship between them.
- Venn diagrams can also be used to show the number of members in a set.
- The **universal set** is the set of elements from which members of all other sets are selected. The symbol $\mathscr{E}$ is used to represent the **universal set**.
- A' is called the **complement** of set A. A' contains all the members of $\mathscr{E}$ that are not in set A.
- The symbol $\varnothing$ is used to represent the **empty set.** $\varnothing = \{\}$
- The symbol $\cap$ is used to represent the **intersection** of two sets. $A \cap B$ is the set of members of $\mathscr{E}$ that are in both set A and set B.



- The symbol $\cup$ is used to represent the **union** of two sets. $A \cup B$ is the set of members of $\mathscr{E}$ that are in set A or in set B or in both sets.



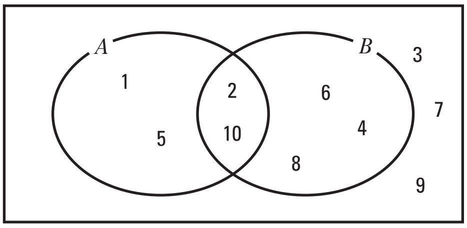
Answers

Chapter 1

M1.1 Get Ready answers

1 even numbers
2 multiples of 5
3 prime numbers

Exercise 1A

1 **a** $A = \{1, 3, 5, 7, 9\}$ **b** $B = \{3, 6, 9, 12\}$
c {3,9} **d** {2, 4, 6, 8, 10, 12}
2 **a** $D = \{2, 3, 5, 7\}$ **b** {2}
c {1, 3, 5, 7, 9}
3


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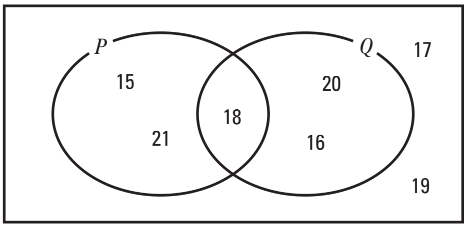

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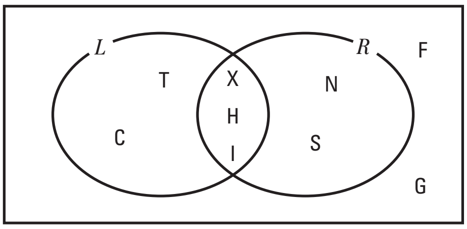

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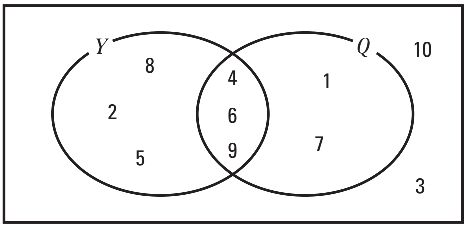
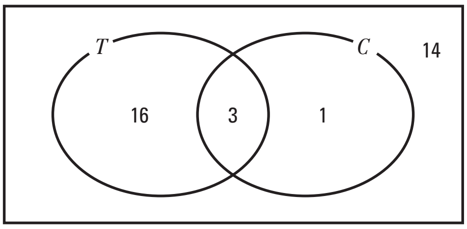
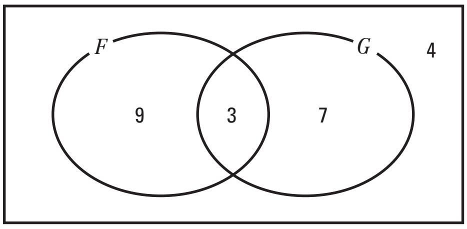
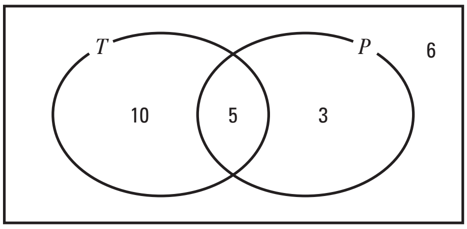


Exercise 1B

1 **a** 35 **b** 6 **c** 9 **d** 7
2 **a**


b 14

3 **a**


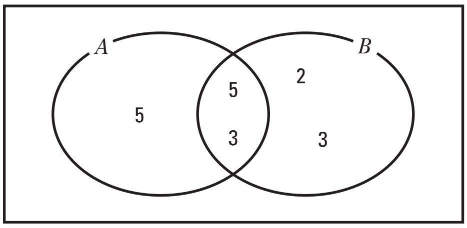
b 7
4


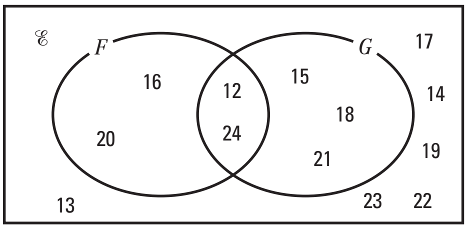
5 11

M1.2 Get Ready answers

1 2, 4, 6
2 1, 3, 5
3 none

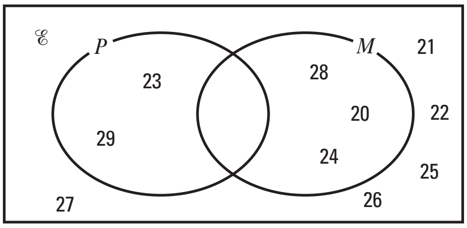
Exercise 1C

1 **a** {2, 5, 7, 8} **b** {2, 5}
c {1, 2, 3, 4, 5, 7, 8}
2 **a** {30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40}
b {30, 32, 34, 36, 38, 40} **c** {31, 32, 34, 35, 37, 38, 40}
d {30, 36}
e {30, 32, 33, 34, 36, 38, 39, 40}
3 **a** 12 **b** 240
4 **a** $75 = 3 \times 5 \times 5$ **b** $90 = 2 \times 3 \times 3 \times 5$
c Venn diagram **d** **i** 15 **ii** 450


5 **a**


b **i** {12, 24}
ii {13, 14, 16, 17, 19, 20, 22, 23}
iii {13, 14, 17, 19, 22, 23}

6 **a**



b **i** ∅ **ii** {20, 24, 28}

7 **a** {7, 10, 11, 13} **b** {8, 9, 12, 14}

c {5, 6, 8, 9, 10, 11, 12, 13, 14, 15}

d {10, 11, 13}

8 **a** **i** {1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19}

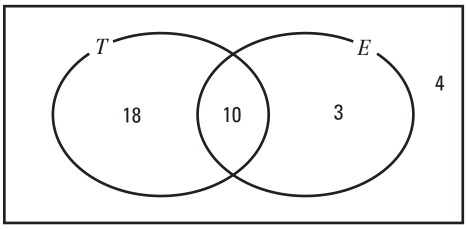
ii {3, 5, 6, 9, 10, 12, 15, 18}

iii {3, 6, 9, 12, 18}

b common multiples of 3 and 5

Exercise 1D

1 **a**



b 10

2 17

3 10

4 29

5 30

M2.1 Quadratic Sequences

Before you start

You should be able to:

- derive and use an expression for the nth term of an arithmetic sequence
- evaluate a quadratic expression given a positive value of the variable.

Why do this?

Quadratic sequences are used when solving the complex equations which describe how weather systems evolve.

Objective

- Derive and use an expression for the nth term of a quadratic sequence.

Get Ready

1 Find an expression in terms of n for the nth term of this arithmetic sequence:
3 5 7 9

2 Work out the value of $5n^2$ when $n = 2$.

3 Work out the value of $n^2 - 3n - 5$ when $n = 3$.

Key Points

- The nth term of a quadratic sequence has the form $an^2 + bn + c$ where a, b and c are numbers.
- The second differences of the terms in a quadratic sequence are constant and equal to $2a$.
- The sequence of square numbers starts 1, 4, 9, 16…
- The nth term of this sequence is n^2. This is the simplest quadratic sequence.

Example 1

The first 4 terms of a quadratic sequence are: 2 5 10 17

Find an expression in terms of n for the nth term.

As a first step, compare this sequence to 1 4 9 16

Comparing to 1 4 9 16 the terms of the quadratic sequence are one more.
The required expression for the nth term is $n^2 + 1$.

Exercise 2A

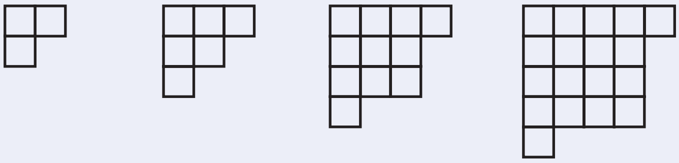
C

1 Find an expression in terms of n for the nth term of the quadratic sequences which start:

a	2	8	18	32
b	0	3	8	15
c	4	7	12	19
d	0	1	4	9
e	1	$1 + 3$	$1 + 3 + 5$	$1 + 3 + 5 + 7$
f	1×2	2×3	3×4	4×5

C

2 Here is a pattern made from centimetre squares.



Pattern 1 Pattern 2 Pattern 3 Pattern 4

a Write down an expression in terms of n for the number of centimetre squares in pattern n.

b Is there a pattern in the sequence which has 170 centimetre squares?
Give a reason for your answer.

3 **a** Find an expression in terms of n for the nth term of the arithmetic sequence:

3 5 7 9

The sequence:

9 25 49 81

is obtained from squaring each term of the arithmetic sequence.

b Find an expression in terms of n for the nth term of this sequence.

B

4 **a** Write down an expression for the nth term of the sequence:

1 4 7 10

b Write down an expression for the nth term of the sequence.

1 16 49 100

c Write down an expression for the nth term of the sequence:

2 18 52 104

Example 2 Find an expression in terms of n for the nth term of the quadratic sequence:

	-1		1		7		17		31
first differences:		2		6		10		14	
second differences:			4		4		4		

Work out the first differences $1 - -1 = 2$, $7 - 1 = 6$, $17 - 7 = 10$ and so on.

Then work out the second differences (the differences of the first differences): $6 - 2 = 4$, $10 - 6 = 4$, $14 - 10 = 4$

If the second differences are constant then the sequence is quadratic.

The expression for the nth term will be of the form $an^2 + bn + c$ where a, b and c are numbers.

a = Second difference $\div 2 = 2$, so the expression for the nth term is $2n^2 + bn + c$

To find the values of b and c, work out the difference between the terms of the sequence and $2n^2$ as shown in the table.

n	1	2	3	4	5
Term	−1	1	7	17	31
$2n^2$	2	8	18	32	50
Term $- 2n^2$	−3	−7	−11	−15	−19

These are the terms of the quadratic sequence

$31 - 2 \times 5^2 = -19$

The sequence −3 −7 −11 −15 −19 has nth term $-4n + 1$.
So the required expression for the nth term of the sequence −1 1 7 17 31 is $2n^2 - 4n + 1$.

Exercise 2B

1 Find the next term and an expression for the nth term of these quadratic sequences:

a	2	6	12	20	30
b	0	2	6	12	20
c	3	7	13	21	31
d	13	17	23	31	41
e	−4	0	10	26	48
f	3	5	8	12	17

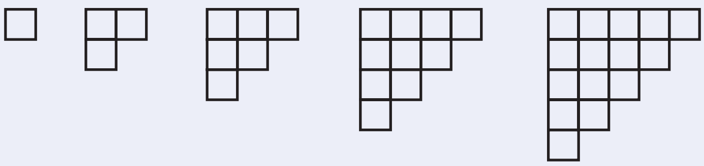
2 Show that 862 is the 20th term of the quadratic sequence:
7 16 29 46 67

3 Show that 5005 is the 50th term of the quadratic sequence:
7 13 23 37 55

4 Here are the first 5 terms of a quadratic sequence:
4 15 30 49 72
Show that there are no prime numbers in the quadratic sequence.

5 Here are the first 4 terms of a quadratic sequence:
3 9 17 27 39
Jim says that 161 is a term of this sequence.
a Is Jim correct? Give a reason for your answer.
Lizzie says that all of the terms are odd numbers.
b Is Lizzie correct? Give a reason for your answer.

6 Here is a sequence of patterns made of centimetre squares.



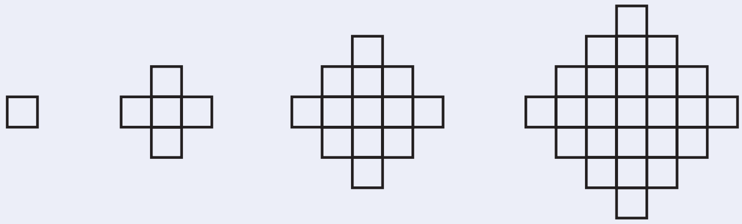
Find the number of centimetre squares in the 100th pattern.

B

A

A

7 Here is a sequence of patterns made of centimetre squares.



Find an expression in terms of n for the number of centimetre squares in the n^{th} pattern.

8 The 4^{th}, 5^{th}, 6^{th} and 7^{th} terms of a quadratic sequence are:

34 37 42 49

Find an expression, in terms of n, for the nth term of this sequence.

9 Here are the first four terms of a quadratic sequence:

4 9 18 x

a Find the value of x.

b Find an expression for the nth term of the sequence.

10 At a party, everyone shakes hands with everyone else. So when there are 4 people at a party there are 6 handshakes. Find an expression in terms of n for the number of handshakes when there are n people at the party.

Review

- The sequence of square numbers begins 1, 4, 9, 16, 25 and the nth term is n^2.
- The nth term of a quadratic sequence can be written as $an^2 + bn + c$.
- The second differences of a quadratic sequence are constant and equal to $2a$.

Answers

Chapter 2

M2.1 Get Ready answers

1 $2n + 1$
2 20
3 -5

Exercise 2A answers

1 **a** $2n^2$ **b** $n^2 - 1$ **c** $n^2 + 3$
d $(n - 1)^2$ **e** n^2 **f** $n(n + 1)$
2 **a** $n^2 + 2$
b No, because $12^2 + 2 = 146$ and $13^2 + 2 = 171$
3 **a** $2n + 1$ **b** $(2n + 1)^2$
4 **a** $3n - 2$ **b** $(3n - 2)^2$ **c** $(3n - 2)^2 + n$

Exercise 2B answers

1 **a** 42, $n^2 + n$ **b** 30, $n^2 - n$
c 43, $n^2 + n + 1$ **d** 53, $n^2 + n + 11$
e 76, $3n^2 - 5n - 2$ **f** 23, $\frac{1}{2}(n^2 + n + 4)$
2 nth term is $2n^2 + 3n + 2$.
When $n = 20$, $2n^2 + 3n + 2 = 800 + 60 + 2 = 862$
3 nth term is $2n^2 + 5$.
When $n = 50$, $2n^2 + 5 = 2 \times 2500 + 5 = 5005$
4 nth term is $2n^2 + 5n - 3$ which factorises to $(2n - 1)(n + 3)$ so cannot be prime
5 **a** nth term is $n^2 + 3n - 1$, 11th term is 153, 12th term is 179, so no.
b Yes, because the first differences are even numbers and the first term is an odd number.
6 nth term $= \frac{1}{2}n^2 + \frac{1}{2}n$, 100th term $= 5050$
7 $2n^2 - 2n + 1$
8 $n^2 - 6n + 42$
9 **a** 31 **b** $2n^2 - n + 3$
10 $\frac{n^2}{2} - \frac{n}{2}$

M4.1 Probability and Venn diagrams

Before you start

You should be able to:

- draw and interpret Venn diagrams
- find the probability that an event will occur.

Why do this?

Venn diagrams can be used to help work out probabilities.

Objective

- You will be able to use set notation to describe events.
- You will be able to use Venn diagrams to find probabilities.

Get Ready

A bag contains 3 red, 2 blue and 6 green counters. A counter is taken at random. What is the probability that the counter is:

1 red **2** green **3** not green
4 blue or green **5** white

Key Points

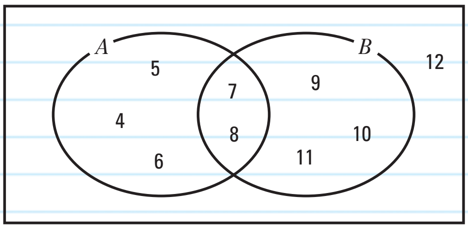
When working out probabilities from a Venn diagram:

- P(A) represents the probability that the item is in set A
- P(A') represents the probability that the item is *not* in set A
- $P(A') = 1 - P(A)$
- P($A \cap B$) represents the probability that the item is in both set A and set B
- P($A \cup B$) represents the probability that the item is in set A or in set B or in both sets.

Example 1

A03

The Venn diagram shows the integers from 4 to 12.



A number is taken at random from those shown on the Venn diagram.

Find: **a** P(A) **b** P(A') **c** P($A \cap B$).

a $P(A) = \frac{5}{9}$

There are 9 numbers in total in the Venn diagram so 9 goes on the bottom of the fraction.
There are 5 numbers altogether in set A so 5 goes on the top of the fraction.

b $P(A') = \frac{4}{9}$

There are 4 numbers that are not in set A.
Alternatively, work out $1 - \frac{5}{9}$

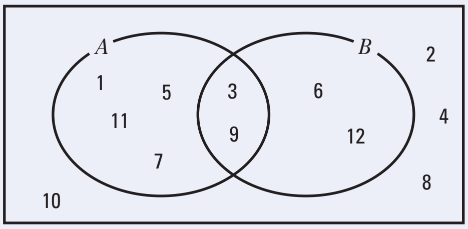
c $P(A \cap B) = \frac{2}{9}$

There are 2 numbers in both A and B.

Exercise 4A

D A01

1 The Venn diagram shows the whole numbers from 1 to 12.

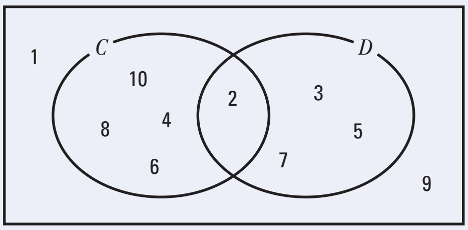


A number is chosen at random from those shown on the Venn diagram.

Find: **a** P(B) **b** P($A \cap B$) **c** P($A \cup B$)

A01

2 The Venn diagram shows the whole numbers from 1 to 10.



A number is chosen at random from those shown on the Venn diagram.

Find: **a** P(D) **b** P(D') **c** P($C \cap D$) **d** P($C \cup D$)

C A03

3 **a** On a Venn diagram show:
the whole numbers from 1 to 12
set E where $E = \{2, 4, 6, 8, 10, 12\}$
set F where $F = \{1, 2, 3, 4, 6, 12\}$

b A number is chosen at random from those in the Venn diagram. Find:
i P(E) **ii** P(F') **iii** P($E \cap F$) **iv** P($E \cup F$)

B A03

4 **a** Draw a Venn diagram to show:
$\mathscr{E}$ = {integers from 10 to 20}
E = {even numbers}
M = {multiples of 5}

b A number is chosen at random from those in the Venn diagram. Find:
i P(M) **ii** P($E \cap M$) **iii** P($E \cup M$) **iv** P($E' \cap M$) **v** P($E \cap M$)$'$

A A03

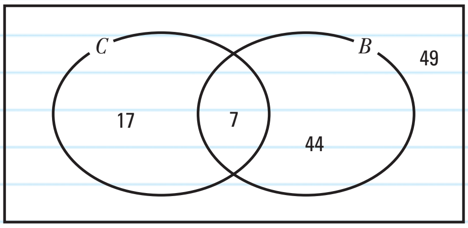
5 $\mathscr{E}$ = {integers from 1 to 20}
M = {multiples of 4}
F = {factors of 20}

A number is chosen at random from the universal set, $\mathscr{E}$.
Work out:

a P(M) **b** P(F') **c** P($M \cap F$) **d** P($M \cup F$) **e** P($M' \cap F$)

Example 2 The Venn diagram shows information about the students in Year 12.

B = {students who take Biology}
C = {students who take Chemistry}



A03

If a student is chosen at random work out:

a P(B) **b** P($B \cap C$) **c** P($C \cap B'$) **d** P($B \cup C$)

a $P(B) = \frac{51}{117}$ — $49 + 44 + 17 + 7 = 117$ there are 117 students in Year 12, so the bottom number of each fraction will be 117. $44 + 7 = 51$ so 51 students study Biology.

b $P(B \cap C) = \frac{7}{117}$ — 7 is in the intersection. This shows that 7 students study Biology and Chemistry.

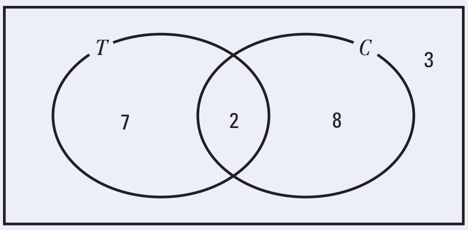
c $P(C \cap B') = \frac{17}{117}$ — There are 17 students who study Chemistry and not Biology.

d $P(B \cup C) = \frac{68}{117}$ — $17 + 7 + 44 = 68$ so 68 students study Biology or Chemistry (or both).

Exercise 4B

1 Some students were asked if they played tennis or cricket.

A01 D



The Venn diagram shows information about their answers.
A student is chosen at random. Work out:

a P(T) **b** P(C) **c** P($T \cap C$)

C A01

2 In a group of 42 people, 13 belong to a badminton club, 19 belong to a tennis club and 7 belong to both a badminton and a tennis club.

- a Draw a Venn diagram to represent this information.
  A person is chosen at random from this group.
- b Find the probability that this person:
  - i does not belong to a badminton club
  - ii does not belong to either a badminton or a tennis club
  - iii belongs to a tennis club but not a badminton club.

B A03

3 There are 26 students in a tutor group. Of these students 11 study History, 17 study PE and 6 students study both History and PE. A student is chosen at random. Work out the probability that this student studies:

a History b PE c History but not PE d neither History nor PE.

A03

4 There are 37 cars parked in a car park. 12 of the cars are red, 22 of the cars are Fords and 8 of the cars are red Fords. One of the cars in the car park is chosen at random. What is the probability that it is:

- a not red
- b a red car that is not a Ford
- c neither red nor a Ford?

A A03

5 There are 29 students in a music class.

13 can play the guitar,
8 can play the piano,
10 cannot play the guitar and cannot play the piano.

One of the 29 students is chosen at random.
Work out the probability that this student can play the guitar but not the piano.

B A03

6 There are 120 people watching a film.

68 have popcorn,
29 have popcorn and a drink,
35 have neither popcorn nor a drink.

One of these people is chosen at random. Work out the probability that this person has a drink but does not have any popcorn.

A03

7 In a group of 35 girls 6 wear glasses, 17 have brown hair and 2 girls have brown hair and wear glasses. One of these girls is chosen at random. Work out the probability that she:

- a has brown hair but does not wear glasses
- b does not have brown hair and does not wear glasses.

M4.2 Compound events

Before you start

You should be able to:

- add and multiply fractions.

Why do this?

Set notation can be used to describe the probability of two events occurring at the same time.

Objectives

- You will be able to use set notation to describe compound events.

Get Ready

1. A fair dice is thrown. Work out the probability of throwing:
   a a 1 or a 2 **b** either an even number or a prime number.
2. Two fair dice are thrown together. The scores are added together. Work out the probability of throwing:
   a a total of 3 **b** a total of 7.

Key Points

- Two events are mutually exclusive when they cannot occur at the same time.
  For mutually exclusive events A and B:
  $\text{P}(A \cup B) = \text{P}(A) + \text{P}(B)$
- Two events are independent if one event does not affect the other event.
  For two independent events A and B:
  $\text{P}(A \cap B) = \text{P}(A) \times \text{P}(B)$

Example 3

A03

M and N are mutually exclusive events.

$\text{P}(M) = \frac{4}{9}$ $\text{P}(N) = \frac{1}{3}$

Work out $\text{P}(M \cup N)$.

$$\text{P}(M \cup N) = \frac{4}{9} + \frac{1}{3}$$

M and N are mutually exclusive events. So use $\text{P}(M \cup N) = \text{P}(M) + \text{P}(N)$

$$= \frac{4}{9} + \frac{3}{9}$$

$$= \frac{7}{9}$$

Example 4

A03

A dice and a coin are thrown.

Event F is getting a 5 on the dice. Event H is getting a head on the coin.

Work out:

a $\text{P}(F)$ **b** $\text{P}(H)$ **c** $\text{P}(F \cap H)$.

a $\text{P}(F) = \frac{1}{6}$

b $\text{P}(H) = \frac{1}{2}$

c $\text{P}(F \cap H) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

Throwing a dice and throwing a coin are independent events, since the outcome of one event does not affect the outcome of the other event. So use $\text{P}(A \cap B) = \text{P}(A) \times \text{P}(B)$

Exercise 4C

C

A01

1 A bag contains 5 red, 3 green and 4 yellow counters.

Event R is getting a red counter.

Event G is getting a green counter.

Event Y is getting a yellow counter.

A counter is taken at random from the bag. Work out:

a $P(R)$ **b** $P(G)$ **c** $P(Y)$

d $P(R \cup Y)$ **e** $P(G \cup Y)$

A01

2 A bag contains 3 red and 4 blue counters.

A box contains 2 red and 5 blue counters.

Event A is getting a red counter from the bag.

Event B is getting a red counter from the box.

One counter is taken at random from the bag and another counter is taken at random from the box. Work out:

a $P(A)$ **b** $P(B)$ **c** $P(A \cap B)$

B

A03

3 The events A and B are mutually exclusive.

Given that $P(A) = \frac{1}{3}$ and $P(B) = \frac{5}{8}$ work out:

a $P(A')$ **b** $P(A \cup B)$

A03

4 The events A and B are independent.

Given that $P(A) = \frac{2}{5}$ and $P(B) = \frac{1}{4}$ work out:

a $P(B')$ **b** $P(A \cup B)$

A03

5 The events D and E are mutually exclusive.

Given that $P(D) = \frac{2}{5}$ and $P(D \cup E) = \frac{3}{4}$ work out:

a $P(D')$ **b** $P(E)$

A

A03

6 $P(C) = \frac{1}{4}$, $P(D) = \frac{2}{5}$, $P(C \cap D) = \frac{1}{10}$

Are events C and D independent?

You must give a reason for your answer.

A03

7 $P(E) = \frac{1}{4}$, $P(F) = \frac{2}{5}$, $P(E \cup F) = \frac{7}{10}$

Are events E and F mutually exclusive?

You must give a reason for your answer.

A03

8 The event X and Y are independent.

Given that $P(X) = \frac{3}{8}$ and $P(X \cap Y) = \frac{9}{44}$ work out $P(Y')$.

Review

- $P(A)$ represents the probability that the item is in set A.
- $P(A')$ represents the probability that the item is *not* in set A.
- $P(A') = 1 - P(A)$
- $P(A \cap B)$ represents the probability that the item is in both set A and set B.
- $P(A \cup B)$ represents the probability that the item is in set A or in set B or in both sets.
- Two events are mutually exclusive when they cannot occur at the same time.

  For mutually exclusive events A and B:
  $P(A \cup B) = P(A) + P(B)$
- Two events are independent if one event does not affect the other event.

  For two independent events A and B:
  $P(A \cap B) = P(A) \times P(B)$

Answers

Chapter 4

M4.1 Get Ready answers

1 $\frac{3}{11}$

2 $\frac{6}{11}$

3 $\frac{5}{11}$

4 $\frac{8}{11}$

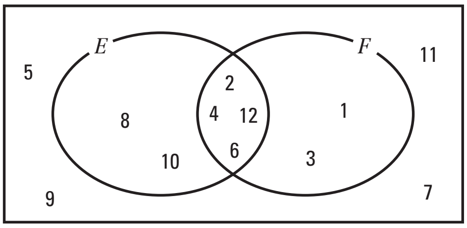
5 0

Exercise 4A

1 **a** $\frac{4}{12}$ **b** $\frac{2}{12}$ **c** $\frac{8}{12}$

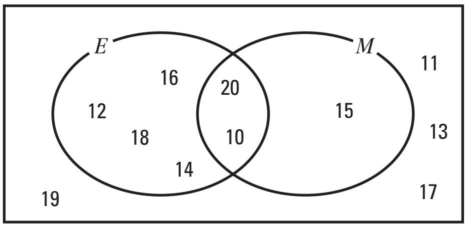
2 **a** $\frac{4}{10}$ **b** $\frac{6}{10}$ **c** $\frac{1}{10}$ **d** $\frac{8}{10}$

3 **a**



b **i** $\frac{1}{2}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{3}$ **iv** $\frac{2}{3}$

4 **a**



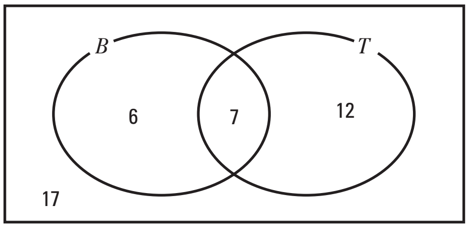
b **i** $\frac{3}{11}$ **ii** $\frac{2}{11}$ **iii** $\frac{7}{11}$ **iv** $\frac{1}{11}$ **v** $\frac{9}{11}$

5 **a** $\frac{1}{4}$ **b** $\frac{14}{20}$ **c** $\frac{1}{10}$
d $\frac{9}{20}$ **e** $\frac{1}{5}$

Exercise 4B

1 **a** $\frac{9}{20}$ **b** $\frac{10}{20}$ **c** $\frac{2}{20}$

2 **a**



b **i** $\frac{29}{42}$ **ii** $\frac{17}{42}$ **iii** $\frac{12}{42}$

3 **a** $\frac{11}{26}$ **b** $\frac{17}{26}$ **c** $\frac{5}{26}$ **d** $\frac{4}{26}$

4 **a** $\frac{25}{37}$ **b** $\frac{4}{37}$ **c** $\frac{11}{37}$

5 $\frac{11}{29}$

6 $\frac{17}{120}$

7 **a** $\frac{15}{35}$ **b** $\frac{14}{35}$

M4.2 Get Ready answers

1 **a** $\frac{1}{3}$ **b** $\frac{5}{6}$

2 **a** $\frac{1}{18}$ **b** $\frac{1}{6}$

Exercise 4C

1 **a** $\frac{5}{12}$ **b** $\frac{1}{4}$ **c** $\frac{1}{3}$
d $\frac{3}{4}$ **e** $\frac{7}{12}$

2 **a** $\frac{3}{7}$ **b** $\frac{2}{7}$ **c** $\frac{6}{49}$

3 **a** $\frac{2}{3}$ **b** $\frac{23}{24}$

4 **a** $\frac{3}{4}$ **b** $\frac{11}{20}$

5 **a** $\frac{3}{5}$ **b** $\frac{7}{20}$

6 Yes as $\frac{1}{4} \times \frac{2}{5} = \frac{1}{10}$

7 No as $\frac{1}{4} + \frac{2}{5} = \frac{13}{20}$

8 $P(Y') = \frac{5}{11}$

A5.1 Exponential growth and decay

Before you start

You need to be able to:

- work out scale factors for percentage increase and decrease.
- draw a graph given y in terms of x.

Why do this?

In science, population growth is often an exponential function of time. The size of an investment in a bank account will grow exponentially if the interest rate remains constant. Radioactive decay is an example of exponential decay.

Objectives

- You can understand the meaning of exponential growth and decay.
- You can use multipliers to explore exponential growth and decay.
- You can use exponential growth in real life problems.

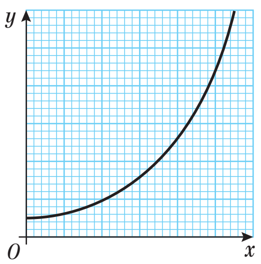
Get Ready

Work out:

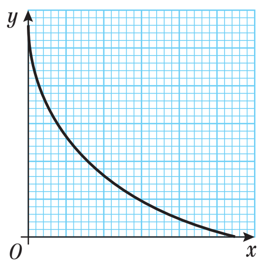
1 5^4 **2** 2^7 **3** 0.8^2 **4** 56^0

Key Points

- Exponential growth occurs when a function keeps increasing by the same scale factor. For example, the size of a population of bacteria may double every hour, the amount invested in a bank account will increase each year by the same scale factor as long as the interest rate remains constant.
- Exponential decay occurs when a function keeps decreasing by the same scale factor. For example, the mass of a radioactive element may halve every hour.
- All exponential growth and decay functions can be represented by the equation $y = ka^x$
- For exponential growth, $a > 1$
- For exponential decay, $0 < a < 1$
- The value of a, called the multiplier, is the scale factor by which the function grows or decays.
- y represents the size of the population or amount at time x
- k represents the initial value of y



$y = ka^x$ $(a > 0)$



$y = ka^x$ $(0 < a < 1)$

AO3

Example 1

A scientist is studying a population of flies.

The size of the population, P, after t days is given by the equation $P = 60 \times 2^t$.

a Work out the size of the population of flies at the beginning of the study.

b How many flies will there be after 10 days?

c Draw a graph to show the size of the population for the first 5 days of the study.

d What happens to the size of the population every day?

a $P = 60 \times 2^0$
$= 60$

At the beginning of the study $t = 0$ so substitute this into the equation.

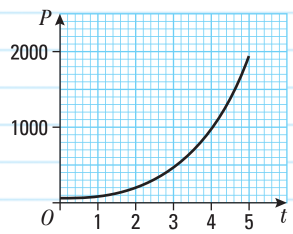
b $P = 60 \times 2^{10}$
$= 61440$

Substitute $t = 10$ into the equation.

c

t	0	1	2	3	4	5
P	60	120	240	480	960	1920

Work out the size of the population for the first 5 days. Use a table to organise your results.



Plot your results on a graph.

d $P = 60 \times 2^t$

The multiplier is 2.

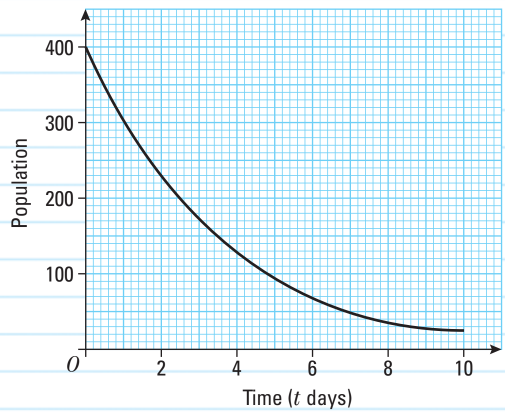
The population doubles every day.

Compare the equation with $y = ka^x$

AO3

Example 2

A scientist recorded the size of a population of small mammals for a number of days. The mammals are suffering from a disease and so the population is decreasing exponentially. The graph shows the size of the population t days after the start of the experiment.



a How many small mammals were present at the beginning of the experiment?

b Work out the decrease in the number of small mammals during the 2nd day of the experiment.

c Work out the percentage change in the number of small mammals each day.

d Work out the multiplier.

a 400 insects ← Read the value of P from the graph when $t = 0$.

b When $t = 1, P = 300$.
When $t = 2, P = 225$. ← The 2nd day of the experiment is between $t = 1$ and $t = 2$. Take readings from the graph at these two values.

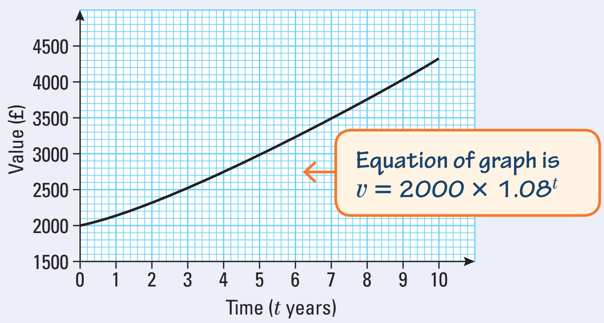
Decrease in number $= 300 - 225$
$= 75$ small mammals ← Subtract these values to find the decrease in the number of small mammals.

c Percentage change $= \frac{75}{300} \times 100$
$= 25\%$ decrease ← As this is an exponential relationship, the percentage change will be the same each day.

d Multiplier $= 100\% - 25\%$
$= 75\%$
$= 0.75$

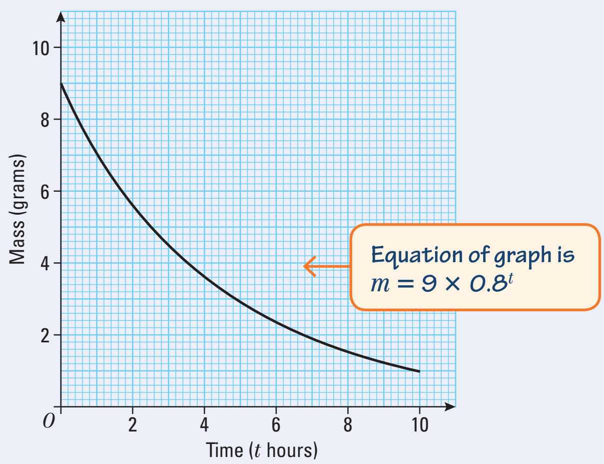
Exercise 5A

1 A03 The graph shows the value, v, of an investment t years after the original amount was invested. The value of the investment increases exponentially.



a What was the original amount invested?
b How much did the investment grow by in the 4th year?
c **i** Work out the multiplier.
ii Work out the interest rate paid.

2 A03 The mass, m grams, of a radioactive substance decreases exponentially as shown in this graph.



a Work out the original mass of the substance.
b Work out the mass of the substance after 6 hours.
c **i** Work out the multiplier.
ii Work out the percentage rate of decrease.

3 A03 The value of a car, £C, t years after the car was bought is given by the equation:
$C = 30\,000 \times 0.7^t$

a Work out the original price paid for the car.
b Draw a graph to show the value of the car for the first five years after the car was bought.
c By what percentage does the price of the car decrease every year?

A* A03

4 The values in the table show the size of a population that is known to be increasing exponentially.

Year	2005	2006	2007	2008	2009
Size of population	43 600	48 832	54 692	61 255	68 605

a Work out the multiplier.

b Work out the likely size of the population in 2015.

Key Points

- An alternative form of the equation $y = ka^x$ is: $A = P\left(\frac{100 + r}{100}\right)^n$

  where: P is the original population (or amount)

  n is the number of years (or hours etc)

  r is the percentage by which the population is increasing (or decreasing)

  A is the population (or amount) after n years.

A01

Example 3

An initial investment of £P grows exponentially at a rate of r% per year.
The size of the investment, A, after n years is given by:

$$A = P\left(\frac{100 + r}{100}\right)^n$$

a An investment is worth £11 576.25 after 3 years. Given that the interest rate was 5% per annum, work out the initial value of this investment.

b Harry invests £2000, after 9 years the value of his investment is £2726. Work out the annual interest rate. Give your answer correct to two significant figures.

a $11576.25 = P\left(\frac{100 + 5}{100}\right)^3$ Substitute the information into the equation.

$11576.25 = P \times 1.05^3$ Work out the sum in the brackets.

$P = \frac{11576.25}{1.05^3}$ Rearrange the equation.

$= £10\,000$

b $2726 = 2000\left(\frac{100 + r}{100}\right)^9$ Substitute the information into the equation.

$\sqrt[9]{\frac{2726}{2000}} = \frac{100 + r}{100}$ Rearrange the equation.

$100 \times \sqrt[9]{\frac{2726}{2000}} - 100 = r$

$r = 3.5\%$

A03

Example 4

The population of an island is increasing exponentially. In 2 years the population increased from 6900 to 8400. Assuming that the population continues to increase at the same rate, what is the population of the island likely to be 5 years after the population was 6900?

$8400 = 6900 \times a^2$

As the population is increasing exponentially it will satisfy the equation $y = ka^x$

$\frac{8400}{6900} = a^2$

$a = \sqrt{\frac{8400}{6900}}$

Solve the equation to work out the value of the multiplier a.

$= 1.1033\ldots$

Multiplier is $1.1033\ldots$

$P = 6900 \times (1.1033\ldots)^5$

Use the original equation with the value for the multiplier and $x = 5$ to work out the likely population.

$= 11282.99\ldots$

$= 11\,300$ correct to 3 significant figures

ResultsPlus
Examiner's Tip

The equation $A = P\left(\frac{100 + r}{100}\right)^n$ could also be used to solve this problem. However, as the percentage rate of increase was not required, it is more efficient to use $y = ka^x$

Exercise 5B

A

1 An initial population, P, grows exponentially at a rate of r% per year. A03
The size of the population, A, after n years is given by:

$A = P\left(\frac{100 + r}{100}\right)^n$

a Given that a population is initially 4000 and is growing exponentially at a rate of 7%, find the size of the population after 10 years.

b Another population grows exponentially from 16 500 to 19 000 in 3 years. Work out the percentage rate of growth.

2 The value of a machine in a factory decreases exponentially from its initial value, £P, at a rate of r% per year. The value of the machine, A, after n years is given by: A03

$A = P\left(\frac{100 - r}{100}\right)^n$

a Given that a machine cost £180 000 initially and its value is decreasing by 14% per annum, find the value of the machine after 10 years.

b Another machine is initially worth £78 000; its value has dropped to £49 000 after 4 years. Find its percentage rate of decrease.

3 An initial investment of £P grows exponentially at a rate of r% per year. A03
The size of the investment, A, after n years is given by:

$A = P\left(\frac{100 + r}{100}\right)^n$

a Ali wants to invest £3000 for 5 years. Bank A offers an interest rate of 3.6%. Bank B offers an interest rate of 3.75%. How much more interest will she earn in 5 years if she invests her money in Bank B?

b The value of an investment at another bank doubles in 15 years. Work out the interest rate.

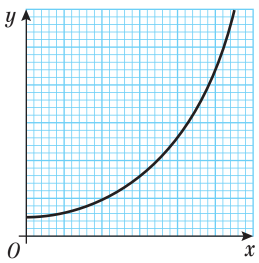
A* A03 **4** The mass, m grams, of a radioactive substance decreases exponentially. It takes 3 days for the mass of the substance to halve. If there is initially 38 grams of the substance, work out how much will remain after 5 days.

A03 **5** The value of an investment is increasing exponentially. In 3 years the value of the investment increases from £15 000 to £18 119. Assuming that the value of the investment continues to increase at the same rate, what is the value likely to be after it has been invested for a total of 8 years?

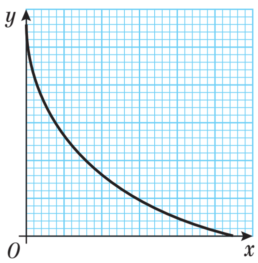
A03 **6** The size of a population is increasing exponentially. Given that it takes 10 years for the population to double, work out the percentage rate at which the population is increasing.

Review

- Exponential growth occurs when a function keeps increasing by the same scale factor. For example, the size of a population of bacteria may double every hour, the amount invested in a bank account will increase each year by the same scale factor as long as the interest rate remains constant.
- Exponential decay occurs when a function keeps decreasing by the same scale factor. For example, the mass of a radioactive element may halve every hour.
- All exponential growth and decay functions can be represented by the equation $y = ka^x$
- For exponential growth, $a > 1$
- For exponential decay, $0 < a < 1$
- The value of a, called the multiplier, is the scale factor by which the function grows or decays.
- y represents the size of the population or amount at time x.
  k represents the initial value of y.



$y = ka^x$ $(a > 0)$



$y = ka^x$ $(0 < a < 1)$

- An alternative form of the equation $y = ka^x$ is: $A = P\left(\frac{100 + r}{100}\right)^n$
  where: P is the original population (or amount)
  n is the number of years (or hours etc)
  r is the percentage by which the population is increasing (or decreasing)
  A is the population (or amount) after n years.

Answers

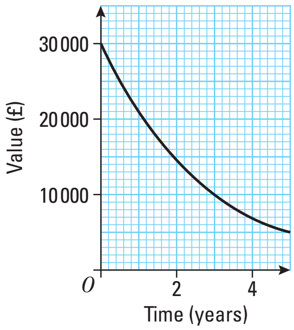
Chapter 5

A5.1 Get Ready answers

1 625
2 128
3 0.64
4 1

Exercise 5A

1 a £2000 b £201.55 c i 1.08 ii 8%
2 a 9 grams b 2.4 grams c i 0.8 ii 20%
3 a £30 000
b



c 30%
4 a 1.12 or 12% b 135 415

Exercise 5B

1 a 7869 b 4.8%
2 a £39 834 b 11%
3 a £25.99 b 4.73%
4 11.97 grams
5 £24 824
6 7.2%

A6.1 AER and compound interest

Before you start

- You should already know how to increase an amount by a given percentage.
- You will need to be able to use your calculator to find the nth root of a number.

Why do this?

Compound interest and the annual equivalent rate (AER) play an important role in everyday investments, especially those taking place over more than two or three years.

Objectives

- Be able to calculate the final amount and the interest on an investment.
- Be able to calculate the annual equivalent rate (AER) of an investment.

Get Ready

1 £6000 is invested at 4% p.a. Work out the value of the investment after one year.

2 Use a calculator to work out:

a 2^{10} **b** $6000 \times \left(1 + \frac{4}{100}\right)^5$

3 Use a calculator to work out $729^{\frac{1}{6}}$

Key Points

- Compound interest is interest paid on the amount and the interest already earned.

Example 1

Katie invests £3000 at 3.4% compound interest.
Work out the value of her investment after 2 years.

Interest after 1 year $= \frac{3000 \times 3.4}{100} = £102$

Value after 1 year = investment + interest after 1 year

Value after 1 year $= 3000 + 102 = £3102$

Interest in year 2 $= \frac{3102 \times 3.4}{100} = £105.47$

Value after 2 years = value after 1 year + interest in year 2

Value after 2 years $= 3102 + 105.47 = £3207.47$

Exercise 6A

C

1 Jim invests £2000 at 3% p.a. compound interest for 2 years. Work out the final amount.

2 Jade invests £1500 at 3% compound interest for 10 years. Work out the final amount.

Key Points

- Compound interest can also be calculated using a formula.
- When £P is invested in an account paying r% compound interest per annum (p. a.), the value, £V, of the investment after n years is given by:

  $V = P\left(1 + \frac{r}{100}\right)^n$
- When £P is invested in an account for n years to produce an investment of value £V, the annual equivalent rate of interest (AER) is given by:

  $\alpha = 100\left(\left(\frac{V}{P}\right)^{\frac{1}{n}} - 1\right)$ where $\left(\frac{V}{P}\right)^{\frac{1}{n}} = \sqrt[n]{\left(\frac{V}{P}\right)}$

Example 2

Katie invests £3000 at 3.4% compound interest.
Work out the value of her investment after 5 years.

$$V = 3000 \times \left(1 + \frac{3.4}{100}\right)^5 = 3000 \times 1.034^5 = £3545.88$$

Substitute $P = 3000$, $r = 3.4$ and $n = 5$ into the compound interest formula.

Example 3

Josh invested £5000 in an account.
After 5 years the value of the account was £7000.
Work out the annual equivalent rate (AER) of the account.

$$\alpha = 100 \times \left(\left(\frac{7000}{5000}\right)^{\frac{1}{5}} - 1\right) = 100 \times \left(1.4^{\frac{1}{5}} - 1\right) = 6.96\%$$

Substitute $P = 5000$, $V = 7000$, and $n = 5$ into the formula

Example 4

Adam invested some money into an account which paid interest annually.
In the first year the account paid 2% compound interest.
In the second year the account paid 4% interest,
and in the third year the account paid 6% interest.
Work out the annual rate of interest (AER) of the account.

$$V = P\left(1 + \frac{2}{100}\right)\left(1 + \frac{4}{100}\right)\left(1 + \frac{6}{100}\right) = 1.124\,448\,P$$

Use the compound interest formula for 3 successive years with the correct value of r each time

Use the AER formula with $n = 3$ and $V = 1.124\,448P$

$$\text{AER} = 100 \times \left(\left(\frac{1.124448\,P}{P}\right)^{\frac{1}{3}} - 1\right) = 100 \times \left(1.124448^{\frac{1}{3}} - 1\right) = 3.99\%$$

NB As a way of checking, Adam's investment should give the same return as if he had invested in an account paying 3.99% p.a. compound interest for 3 years

$$P \times \left(1 + \frac{3.99}{100}\right)^3 = 1.1245\ldots \times P$$

which compares well with the $1.124448P$ above, the difference being due to the rounding of the AER to 2 decimal places.

Example 5

Holly invests £10000 in an account with an annual equivalent rate AER of 5%. She gets the interest paid half yearly. Work out the value of her first half yearly interest payment.
Suppose her half yearly interest payment rate is x%, then:

$$\left(1 + \frac{x}{100}\right)^2 = \left(1 + \frac{5}{100}\right)$$

This line comes from using compound interest for two successive half years and setting it equal to using 5% for 1 year. This line will be true no matter how much money is invested.

So: $1 + \dfrac{x}{100} = \sqrt{1.05}$ $x = 100 \times (\sqrt{1.05} - 1) = 2.469\,5\,\%$

The amount of money added to the account is $£10\,000 \times 2.469\,5\,\% = £246.95$.

Exercise 6B

B

1 Work out the value of these investments in accounts paying annual compound interest after the number of years stated.

	Initial Investment	Annual Interest rate	Number of years
a	£5000	5%	3
b	£2000	4%	5
c	£500	3.5%	6
d	£250	2.8%	10
e	£750	4.7%	18

2 Bill invests £5000 in an account paying 4% compound interest p.a. for 6 years. Work out the total interest that the account earns. A03

3 Mr Smith invests £10 000 in a savings scheme for 6 years. The AER of the savings scheme is 3.2%. A03
Mr Smith will have to pay tax at 40% on the total interest he gets at the end of the 6 years.
Work out how much tax Mr Smith will have to pay on the investment.

4 Every year Jim invests £1000 in an account paying 3% compound interest p. a. Work out the amount of money in the account at the end of the third year. A02

5 Mrs Newton wants to invest some money to pay for her son to attend university. She plans to invest in an account which pays 4.8% per annum compound interest. How much will she have to invest so that the account is worth £6000 after 5 years? A03

6 Ravi has £8000 to invest. He intends to leave it in his account for 6 years. What rate, per annum, of compound interest will enable the value of the account to reach £10 000 after 6 years? A03

7 An account pays 6% compound interest per annum. How many years will the investment have to be in place before its value doubles? A03

8 Work out the annual equivalent rate (AER) for each of these investments. A03

	Initial Investment	Number of years (n)	Value of the investment after n years
a	£5000	3	£6000
b	£2000	5	£2200
c	£500	6	£720
d	£250	10	£318
e	£750	18	£1710.50

A

9 James invests £1000 in an account. For the first year the account paid interest at 5% p.a. For the second year the account paid interest at 3.5% p.a. Work out the annual equivalent rate (AER) of interest on this account. Give your answer correct to 4 significant figures. A02

10 Annette invested £2500 in an account. In the first year the interest rate was 3%, in the second year 5% and in the third year 7%. A03

a Work out the value of Annette's account at the end of 3 years.

b Work out the annual equivalent rate (AER). Give your answer correct to 4 significant figures.

A

A03 **11** Naseem invests £20 000 in an account. For the first two years the account pays 4% per annum compound interest, and for the next three years the account pays 6% per annum compound interest.

a Work out the value of the account after 5 years.

b Work out the annual equivalent rate (AER). Give your answer correct to 4 significant figures.

A02 **12** A savings plan lasts for 5 years. For the first year the interest rate is 2%. The interest rate increases by 1% every year for the life of the savings plan.
Work out the annual equivalent rate (AER). Give your answer correct to 4 significant figures.

A03 **13** An account pays 4% compound interest on the amount in the account every six months. What is the annual equivalent rate of interest?

A6.2 Cost of living index

Before you start

- You should be able to calculate with money.
- You should be able to calculate a percentage of an amount.

Why do this?

Basic money calculations are essential in modern life and having an understanding of the cost of living is useful when judging the value of wage rises.

Objectives

- Gain an understanding of financial mathematics.
- Be able to calculate wage increases which are in line with cost of living increases.

Get Ready

1. Work out 3% of £180.
2. Increase £320 by 5%.

Key Points

- The cost of living index is a measure of how prices increase. It is linked with the idea of inflation of prices.
- The cost of living index has a base year when the index is set equal to 100.
- The cost of living index increases by an amount each year, which depends on the costs of a typical set of items that people buy.

Example 1 Jim pays rent on a flat. Each year the rent increases in line with the cost of living index.

In 2010 the rent was £420 per month and the cost of living index was 100
In 2011 the cost of living index was 103.5
Work out what Jim's rent will be in 2011.

The cost of living increases by 3.5%.
Jim's rent will increase by 3.5% $= 420 \times \frac{3.5}{100} = 14.7$.
Jim's new rent will be £434.70 per month.

The increase in the cost of living is 103.5 − 100 out of 100.

Example 2 Here is Mr Lincoln's bank statement from 1 April to 28 April. Some items are missing.

A deposit in an account happens when money is added to the account.

Date	Deposit (£)	Withdrawal (£)	Balance (£)
1.4.2012			3420.26
6.4.2012		200.00	3220.26
13.4.2012	312.51		3532.77
20.4.2012		250.00	....
28.4.2012	1250.00		....

The balance is the amount of money in the account.

a Write down how much was in the account on 1 April.
b Copy and complete Mr Lincoln's bank account.
c Mr Lincoln wants to know whether he can afford to pay a deposit of £4500 on a car. Can he afford it?

a £3420.26
b Missing items are £3282.77 and £4532.77
c Yes as the balance of his account is more than £4500.

Exercise 6C

F

1 John earns £250. He gets a wage rise of 10%. Work out his new wage.

2 Ben can buy 4 tins of tomatoes at 59p each or he can buy a bargain pack of 4 tins of tomatoes for £1.99. Work out how much he can save. A03

3 A litre of fuel costs 121.9p.
a Lizzie buys 25 litres of fuel. How much will she have to pay?
b Amir buys £40 worth of fuel. How much fuel does he buy?

E

4 Annie's rent is £112 per week. She gets a 10% reduction. Work out her new rent.

5 A student railcard costs £26. The railcard allows a student to buy rail tickets with $\frac{1}{3}$ off the normal price. A03
Anya wants to get a rail ticket. The normal price is £114.
How much money can she save by buying a railcard and using it to reduce the price of the rail ticket?

E

AO2

6 **a** Lethna has £1.80. She wants to buy a drink and fries.
What are the different combinations that can she buy?

b Ken buys:
2 double burgers with cheese, 1 large portion fries and 1 large cola.
He pays with a £10 note. He gets the best price.
What change should he get?

Ben's Burger Bar

Burgers

Single burger	£0.85
Single burger with cheese	£0.95
Double burger	£1.55
Double burger with cheese	£1.70

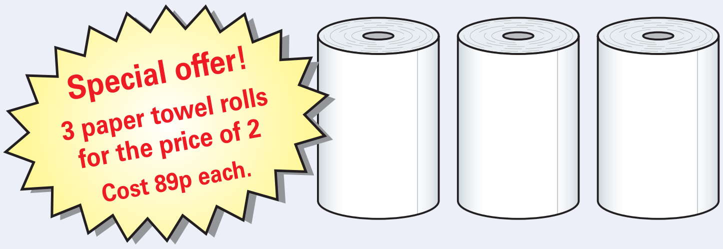
Fries		Cola	
Regular	£0.65	Regular	£0.85
Large	£0.99	Large	£1.10

Meal Deals

Regular	
Single burger with cheese, regular fries and regular cola	£2.09
Large	
Double burger with cheese, large fries and large cola	£3.49

AO3

7



Natasha wants to buy 6 paper towel rolls. Work out how much she can save by using the special offer.

8 Javier gets the bus to and from work each day. He can get a daily return costing £2.90 or he can get a 5-day return costing £12.
How much will he save each week by buying a 5-day return?

9 Fred can buy a season ticket to watch his football team's home games. It will cost him £720 and allows him to attend all his team's home games.
Without a season ticket it will cost Fred £32 to attend each home game.
Fred's football team plays 23 home games.
Work out how much Fred would save by buying a season ticket.

D

10 Saeed earns £18 000 in a year. He does not pay tax on the first £6000 of the £18 000.
He pays tax 20% on the remainder.
Work out how much tax Saeed has to pay.

AO3

11 In 2009 Jenna found she had spent £3000 on rent, £800 on heating and £400 on rates.
In 2010, her rent for the year increased by 5%, heating by 15% and rates by 10%.
Work out the total increase in the amount of money that Jenna spent on these three items in 2010.

12 Oscar buys a car. The cash price of the car is £25 000.
Oscar pays a deposit of 30% of the cash price, followed by 24 monthly payments of £800 each.
How much altogether does Oscar pay for the car?

13 On average the cost of living is 5% higher in Cambridge than in Swindon.
Sophie spends £25 000 each year living in Swindon. How much would it cost her to live in Cambridge?

* **14** Jodie buys a car. The cash price of the car is £24 000.
Jodie pays a deposit of 25%, followed by 24 monthly payments of £900 each.
Bob says that overall Jodie will be paying more than 120% of the cash price of the car.
Is Bob correct? Explain your answer. A02

15 The cost of living index was 100 in 2005. It increased by 3% by the start of 2006.
Leonie gets a pay rise at the start of 2006 in line with inflation.
In 2005 she earned £1400 per month.
How much would she earn each month after her pay rise? A03

16 The cost of living index was 100 in 2005. It increased to 110.8 in 2009.
The living costs of Steve's family increased in line with inflation. In 2005 that cost was £800 per week.
How much was it in 2009?

* **17** The cost of living index was 100 in 2005. It increased to 108.5 in 2008.
The cost of a litre of petrol in 2005 was 88p. The cost of a litre of petrol in 2008 was £1.03.
Did the cost of petrol go up by a bigger percentage than the cost of living?
Explain your answer. A02

18 The cost of living index was 100 in 2005. It increased to 114 in 2010. The national minimum adult wage in 2005 was £5.05 per hour. A03

a What would the national minimum adult wage have to be in 2010 to keep pace with inflation?
The national minimum wage for 16-17 year olds in 2005 was £3.00 per hour. In 2010 it was £3.57.

***b** Has the national minimum wage for 16-17 year olds kept pace with inflation?
Explain your answer.

* **19** In 2007, Fran earned £20 000 per year. She spent 15% of her earnings on rent.
By 2009, Fran's wage had increased by 5%. Her rent was now £3500 per year.
Does Fran spend a greater or smaller percentage of her earnings on rent in 2009 than she did in 2007?
You must give a reason for your answer. A02 A03

20 Rail operators are allowed to raise fares by the cost of living index increase + 1%. In 2010, the cost of living increase was 4.5%.
The fare from Bristol to London in 2010 was £120.
What is the new fare in 2011 assuming the rail operator applies the maximum increase?

C

Review

- When £P is invested in an account paying r% compound interest per annum (p. a.), the value, £V, of investment after n years is given by:
$V = P\left(1 + \frac{r}{100}\right)^n$
- When £P is invested in an account for n years to produce an investment of value £V, the annual equivalent rate of interest (AER) is given by:
$\alpha = 100\left(\left(\frac{V}{P}\right)^{\frac{1}{n}} - 1\right)$ where: $\left(\frac{V}{P}\right)^{\frac{1}{n}} = \sqrt[n]{\left(\frac{V}{P}\right)}$
- The cost of living index gives information about the increase in cost of a set of typical items for a family over one year.

Answers

Chapter 6

A6.1 Get Ready answers

1 £6240
2 a 1024 b 7299.917
3 3

Exercise 6A

1 £2121.80
2 £2015.87

Exercise 6B

1 a £5788.13 b £2433.31 c £614.63
d £329.51 e £1714.36
2 $5000 \times 1.04^6 = £\,6326.60$ Interest = £1326.60
3 $10\,000 \times 1.032^6 - 10\,000 = £2080.31$
Tax = £832.12
4 $1000 \times 1.03^3 + 1000 \times 1.03^2 + 1000 \times 1.03 = £3183.63$
5 $P \times \left(1 + \frac{4.8}{100}\right)^5 = 6000 \quad P = \frac{6000}{1.048^5} = £4746.19$
6 $r = 100 \times \left(\left(\frac{10000}{8000}\right)^{\frac{1}{6}} - 1\right) = 3.79\%$
7 $P \times 1.06^n = P \times 2$
T&I gives $n = 11.9$ so after 12 full years.
8 a $100 \times \left(\left(\frac{6000}{5000}\right)^{\frac{1}{3}} - 1\right) = 6.27\%$
b $100 \times \left(\left(\frac{2200}{2000}\right)^{\frac{1}{5}} - 1\right) = 1.92\%$
c $100 \times \left(1.44^{\frac{1}{6}} - 1\right) = 6.27\%$
d $100 \times \left(1.272^{\frac{1}{10}} - 1\right) = 2.44$
e $100 \times \left(\left(\frac{1710.50}{750}\right)^{\frac{1}{18}} - 1\right) = 4.69\%$
9 $V = 1000 \times 1.05 \times 1.035 = £1086.75$
$\alpha = 100 \times \left(\left(\frac{1086.75}{1000}\right)^{\frac{1}{2}} - 1\right) = 4.247\%$
10 a $2500 \times 1.03 \times 1.05 \times 1.07 = £2893.01$
b $100 \times \left(\left(\frac{2893.10}{2500}\right)^{\frac{1}{3}} - 1\right) = 4.988\%$
11 a $20\,000 \times 1.04^2 \times 1.06^3 = £25\,764.06$
b $100 \times \left(\left(\frac{25\,764.06}{20\,000}\right)^{\frac{1}{5}} - 1\right) = 5.195\%$
12 $V = P \times 1.02 \times 1.03 \times 1.04 \times 1.05 \times 1.06$
$\alpha = 100 \times \left(\left(\frac{1.02 \times 1.03 \times 1.04 \times 1.05 \times 1.06}{1}\right)^{\frac{1}{5}} - 1\right)$
$= 3.990\%$
13 $\alpha = 100 \times (1.04^2 - 1) = 8.16\%$

A6.2 Get Ready answers

1 £5.40
2 £336

Exercise 6C

1 £275
2 236p − 199p = 37p
3 a £30.47 or £30.48 b 32.81 litres
4 £100.80
5 $\frac{114}{3} = 38$ so £12
6 a Regular fries with regular cola, Regular fries with large cola
b £4.81
7 £1.78
8 £2.50
9 £16
10 £2400
11 £150 + £120 + £40 = £310
12 £7500 + £19 200 = £26 700
13 £26 250
14 No. £6000 + £21 600 = £27 600, 1.20 × £24 000 = £28 800
15 £1442
16 £886.40 per week
17 88p × 1.085 = 95.48p which is less than 103p so the price of petrol has risen faster
18 a £5.76
b 300p × 1.14 = 342p so is above inflation.
19 New wage = £21 000.
New percentage = $\frac{3500}{21\,000} \times 100 = 16.7\%$, which is greater than 2007.
20 £126.60

A9.1 Linear programming

Before you start

You should be able to:

- show by shading a region defined by one or more linear inequalities.

Why do this?

Linear programming is an example of optimisation which is very important in manufacturing.

Objectives

- You will be able to find the maximum and minimum value of a linear function within a region in the xy plane.
- You will be able to formulate and solve a linear programming problem in two variables.

Get Ready

1 Draw the lines with equations:
a $y = x + 4$ **b** $2x + 3y = 6$

2 Show by shading on your graph the regions:
a $y \leqslant x + 4$ **b** $2x + 3y \leqslant 6$

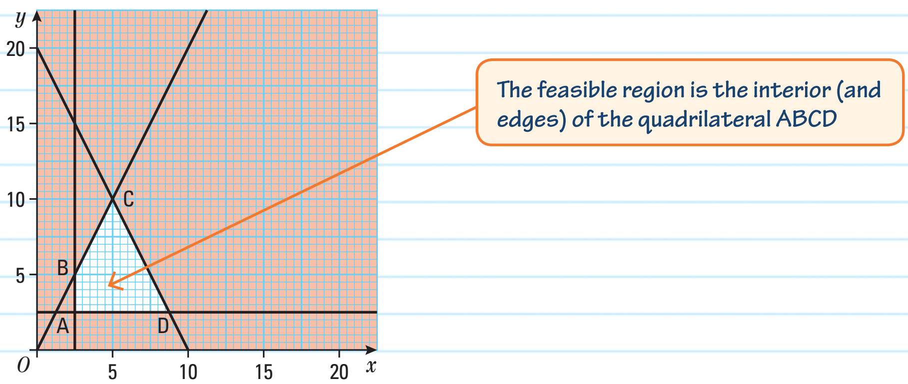
3 Show by shading the region of points which satisfy all of these inequalities:
$x \geqslant 5, y \geqslant 6$ and $x + y \leqslant 13$

Key Points

- A set of linear inequalities of the form $ax + by \leqslant c$ can define an enclosed region, R, known as the feasible region.
- The coordinates of all the points within and on the boundaries of the feasible region satisfy all the inequalities.
- A linear function P is of the form $P = ax + by + c$ where a, b and c are numbers.
- Within an enclosed region R, the maximum and minimum values of any linear function are attained at one of the corners of the region or along an edge of the region.
- Note: it is easier to show the region which satisfies all the inequalities as unshaded.

Example 1

The feasible region ABCD shown (unshaded) below satisfies these linear inequalities:
$2x + y \leqslant 20$, $x \geqslant 2$, $y \geqslant 2$ and $y \leqslant 2x$



Find the maximum and minimum values of the following linear functions:

a $P = 4x + 3y$ **b** $P = 4x + 2y$

a So the maximum value of P (50) is at D
and the minimum value (14) is at A.

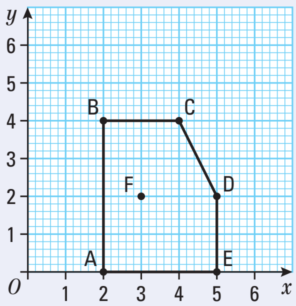
b At each of the corners, P takes the values A 12, B 16, C 40, D 40
So the maximum value occurs (40) at the Points C and D
(and in fact anywhere along the edge CD).
The minimum value (12) occurs at A.

(a) At the corner A (2, 2), P takes the value $4 \times 2 + 3 \times 2 = 14$.
At the corner B (2, 4), P takes the value $4 \times 2 + 3 \times 4 = 20$.
At the other corners P takes the values (C), 35 and (D), 42

Exercise 9A

C

1



Find the value of each of these linear functions at the point A, B, C, D, E and F as shown in the diagram.

a $2x$ b $3y$ c $x + y$ d $2x + y$
e $3x - y$ f $x + 2y + 3$

B

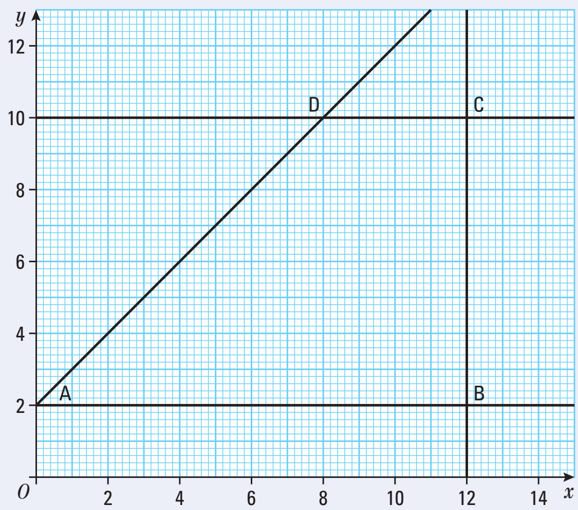
2 Draw the region which satisfies all of the following inequalities:
$x \geqslant 2,\ y \geqslant 3, x + 2y \leqslant 12$
Find the value of each of these linear functions at the vertices of the region and at the point with coordinates (3, 4):

a $2x$ b $x + y$ c $2y - 3$ d $3x + y$ e $x - 2y$

A

3 The diagram shows the finite region ABCD.
Write down the equations of each of the boundary lines of ABCD.
Find the inequalities that points within or on the boundary of ABCD must satisfy.
Work out the maximum and minimum values of the following functions at points within or on the boundary of ABCD:

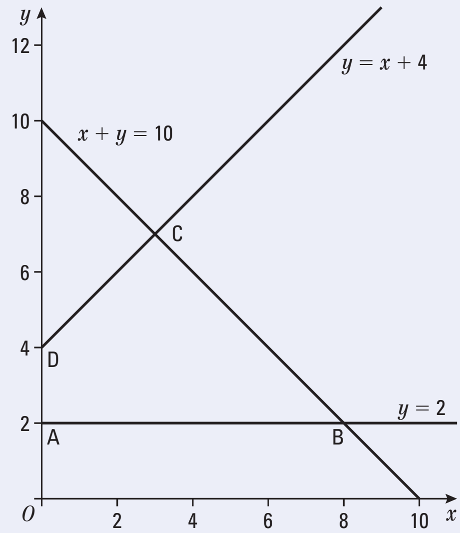
a $x + y$ b $2x + 3y$
c $x - y$ d $2x - y + 8$



A

4 Here is a sketch.
Find the maximum and minimum values taken by each of these functions within or on the boundary of the finite region ABCD.

a $x + 2y$ **b** $2x + y$
c $4x + 4y$ **d** $2y - 2x$



5 The region ABCD satisfies the following inequalities:
$y + 3x \leqslant 48$ $y \geqslant x + 4$ $y \leqslant 12$ $4y + x \geqslant 16$.
Show, by shading, the region ABCD.

Find the maximum and minimum values of the following functions which satisfy all of the above inequalities.

a $x + 2y$ **b** $2y - x$ **c** $4x + y$ **d** $6x + 2y$

6 Show, by shading, the region which satisfies all of the following inequalities:
$y \leqslant x$ $2y + x \leqslant 150$ $y + x \leqslant 130$ $8y \leqslant 3x + 50$ $y \geqslant 0$
Find the maximum and minimum values of each of the following functions for the set of points which satisfy all of the above inequalities.

a $x + y$ **b** $2x - y$ **c** $3x + 2y + 50$ **d** $2x + 3y + 40$

Example 2 A radio broadcast company has two stations: Hot Hits and Cool Classics.

The company spends daily at least twice as much on Hot Hits as on Cool Classics.
The company spends daily at least £1000 on Cool Classics and at least £4000 on Hot Hits.
The company can afford to spend daily no more than a total of £12 000.
Let £x be the money the company spends daily on Hot Hits.
Let £y be the money the company spends daily on Cool Classics.

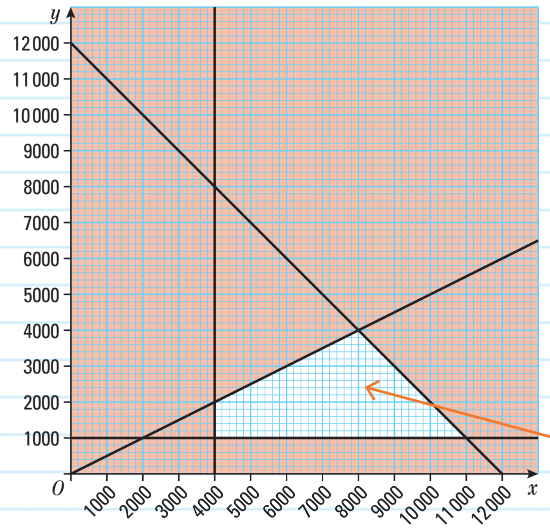
a Write down 4 constraints that x and/or y must satisfy.
b Draw a suitable diagram and identify the region that satisfies all of the constraints.

The daily profit on Cool Classics is expected to be £30 per pound spent and the daily profit on Hot Hits is expected to be £15 per pound spent.

c Write down an expression for the total daily profit £P in terms of x and y.
d Use your diagram to find the maximum daily profit and the values of x and y at which it occurs.

a $x \geqslant 2y \quad x \geqslant 4000 \quad y \geqslant 1000 \quad x + y \leqslant 12000$

b



A constraint is a mathematical condition that must be satisfied. In linear programming the constraints can be written as linear inequalities

ResultsPlus
Examiner's Tip

Candidates often write $x \geqslant 2y$ the wrong way round as $x \geqslant \frac{1}{2}y$.

This is the feasible region – it satisfies all four of the constraints.

c $P = 15x + 30y$

d Maximum profit occurs at (8000, 4000) and is £240 000.

Exercise 9B

A*

1 A farmer puts fertiliser on his fields.
He knows he must put on at least 500 kg of phosphate and at least 800 kg of nitrate.
The maximum total amount of fertiliser he will put on his fields is 3000 kg.
Let x kg be the mass of phosphate.
Let y kg be the mass of nitrate.

a Write down 3 constraints that x and/or y must satisfy.

b Draw a graph and indicate on the graph the region which satisfies all 3 constraints.

The cost of one kg of phosphate is 30p.
The cost of one kg of nitrate is 20p.

c Write down an expression for the cost C pence of x kg of phosphate and y kg of nitrate.

d Find the minimum and maximum cost which satisfies all the constraints.

2 A company makes shirts and vests.

Each day the company must make at least 300 shirts and must make at least 200 vests.
The company makes at least as many shirts as vests each day.
The company can make a maximum of 1000 of these garments each day.
Let x be the number of shirts.
Let y be the number of vests.

a Write down the inequalities that x and/or y must satisfy.

b On graph paper, show by shading, the region which satisfies all of the inequalities.

The cost of making a shirt is £20.
The cost of making a vest is £30.

c Write an expression for the total cost £C of making x shirts and y vests.

d Work out the minimum cost that satisfies all the constraints.

e The profit on a shirt is £10 and the profit on a vest is £5. Assuming that the company sells all the articles it makes, work out the maximum profit.

A*

3 A company makes chairs and settees.
Every day the company can make a maximum of 600 pieces of furniture.
The company makes at most twice as many chairs as settees.
The company makes at least 100 chairs and at most 300 settees each day.
The cost of making a chair is £100 and the cost of making a settee is £160.

Let x be the number of chairs and y be the number of settees.

a Express each of the constraints as inequalities.
b Express the total cost in terms of x and y.
c Draw the feasible region on a grid of squares.
d Find the minimum and maximum costs and the number of chairs and the number of settees at which the minimum cost is attained.

4 A market gardener grows cabbages and carrots.
She has a maximum of 80 hectares for growing.
She grows carrots on at least 50% more land than she grows cabbages.
She must use at least 20 hectares for carrots and at least 10 hectares for cabbages.

Let x hectares be the area used for growing carrots and y hectares be the area used for growing cabbages.

a Write down the inequalities.

The revenue from a hectare of carrots is £300 and the revenue from a hectare of cabbages is £400.

b Write down an expression in terms of x and y for the total revenue, £R.
c Find the maximum value of the revenue £R.

5 A newspaper runs a lottery in which there are £10 prizes and £20 prizes.

There must be at least 12 £20 prizes.
There must be at least 20 £10 prizes.
The number of £20 prizes must be not be more than 16 more than the number of £10 prizes.
The total amount of money available for the prize fund must not be greater than £1000.

Let x be the number of £10 prizes and y be the number of £20 prizes.

a **i** Explain why $x + 2y \leqslant 100$
ii Write the other constraints as inequalities.
b Draw these inequalities on graph paper and identify the region that satisfies all the inequalities.
c What is the maximum total number of prizes that can be given?

6 Tickets at a concert cost either £20 or £50.

The number of £50 tickets must be no more than 200 more than the number of £20 tickets.
There must be at least 300 £50 tickets and at most 600 £20 tickets.
The total number of tickets must not be more than 1200.

Let x be the number of £50 tickets and let y be the number of £20 tickets.

a Write these constraints as inequalities.
b Draw these inequalities on a suitable grid.

The profit from each £50 ticket is £20 and the profit from each £20 ticket is £10.

c Write down an expression for the total profit, £P.
d Find the maximum profit.

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A*

7 Bill takes lots of exercise. Each week he covers between 40 miles and 80 miles by a combination of walking and jogging.
He walks at most half as far as he jogs.
He walks a minimum of 16 miles, and jogs a minimum of 20 miles.
Let x miles be the distance he walks and let y miles be the distance he jogs.

- **a** Write down 5 relevant constraints that x and y must satisfy.
- **b** Draw a graph to show the region of points satisfied by the constraints.

Bill uses up 150 calories per mile when he walks and 250 calories per mile when he jogs.

- **c** Use your graph to find the smallest number and the largest number of calories that Bill can use up each week through this exercise.

8 A company makes two types of phones, A and B.
Each day it must make at least 200 type A and at least 300 type B.
The number of type B must be at most 50% more than type A.
The total number of phones made each day must not be more than 1000 and must not be less than 600.
Let x be the number of type A phones.
Let y be the number of type B phones.

- **a** Write down all the constraints as inequalities.
- **b** Show by shading the region which satisfies all the constraints.

The profit from making a type A is £6. The profit from making a type B is £7.50.

- **c** Assuming that the company sells all the phones it makes, work out the maximum profit from the day.

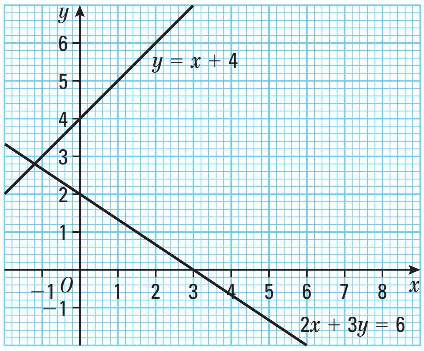
Review

- The solution to a linear programming problem requires the evaluation of a linear function at the corners of the feasible region.
- The maximum (minimum) value of a linear function in an enclosed region defined by a set of linear inequalities occurs at one of the corners of the region or at all the points along the edge of the region.

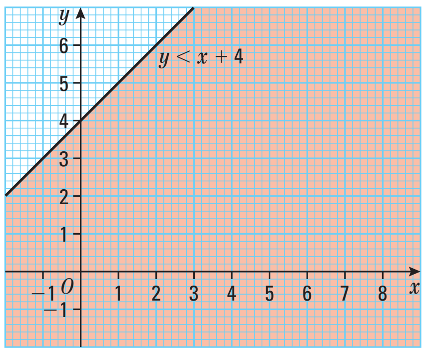
Answers

A9.1 Get Ready answers

1



2 a



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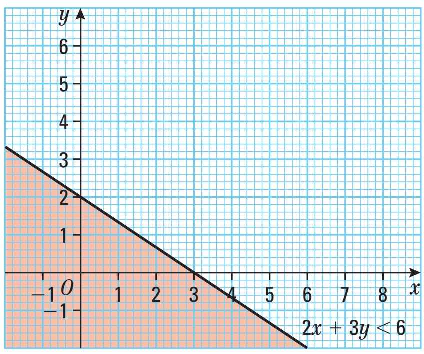

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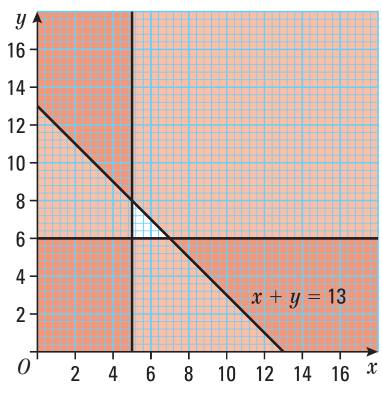
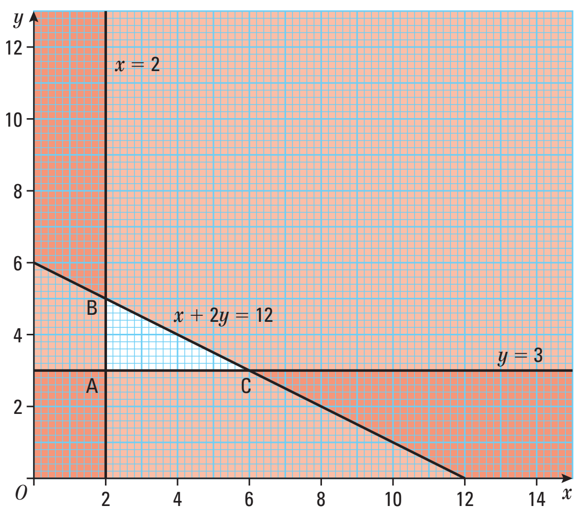


Exercise 9A

1

	A	B	C	D	E	F
$2x$	4	4	8	10	10	6
$3y$	0	12	12	6	0	6
$x + y$	2	6	8	7	5	5
$2x + y$	4	8	12	12	10	8
$3x - y$	6	2	8	13	15	7
$x + 2y + 3$	5	13	15	12	8	10

2



	A	B	C	(3, 4)
$2x$	4	4	12	6
$x + y$	5	7	9	7
$2y - 3$	3	7	3	5
$3x + y$	9	11	21	13
$x - 2y$	−4	−8	0	−5

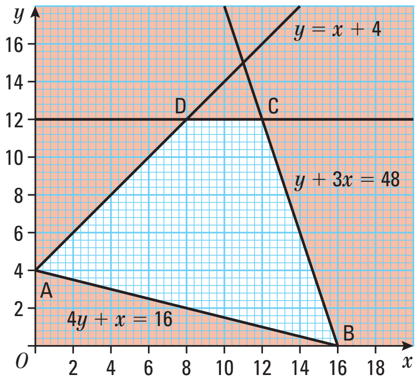
3 Through AB $y = 2$
Through BC $x = 12$
Through AD $y = x + 2$
Through DC $y = 10$
Inequalities satisfied are
$y \geqslant 2 \quad x \leqslant 12 \quad y \leqslant 10 \quad y \leqslant x + 2$

	A	B	C	D
$x + y$	2 (min)	14	22 (max)	18
$2x + 3y$	6 (min)	30	54 (max)	46
$x - y$	−2 (min)	10 (max)	2	−2 (min)
$2x - y + 8$	6 (min)	30 (max)	22	14

4

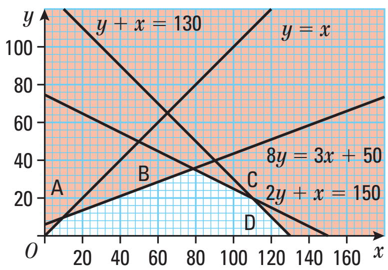
	A	B	C	D
$x + 2y$	4 (min)	12	17 (max)	8
$2x + y$	2 (min)	18 (max)	13	4
$4x + 4y$	8 (min)	40 (max)	40 (max)	16
$2y - 2x$	4	−12 (min)	8 (max)	8 (max)

5



	A	B	C	D
$x + 2y$	8 (min)	16	36 (max)	32
$2y - x$	8	−16 (min)	12	16 (max)
$4x + y$	4 (min)	64 (max)	60	44
$6x + 2y$	8 (min)	96 (max)	96 (max)	72

6

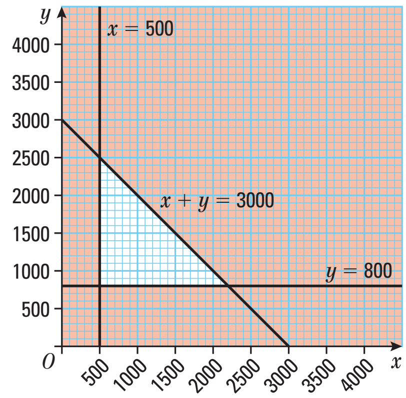


	O	A	B	C	D
$x + y$	0 (min)	20	114	130 (max)	130 (max)
$2x - y$	0 (min)	10	120	200	260 (max)
$3x + 2y + 50$	50 (min)	100	356	420	440 (max)
$2x + 3y + 40$	40 (min)	90	304	320 (max)	300

Exercise 9B answers

1 a $x \geqslant 500$, $y \geqslant 800$, $x + y \leqslant 3000$

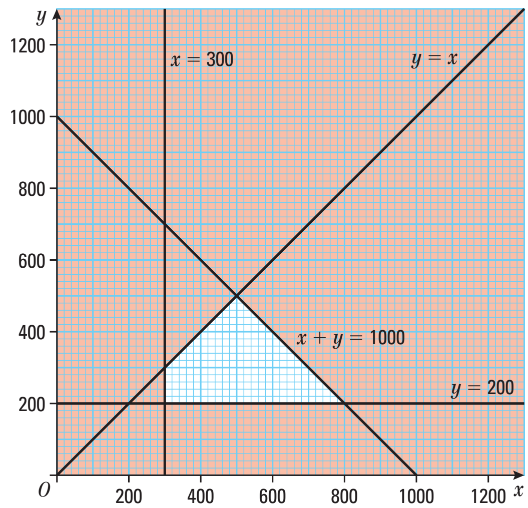
b



c $C = 30x + 20y$

d min = £310, max = £820

2 a $x \geqslant 300$, $y \geqslant 200$, $y \leqslant x$, $x + y \leqslant 1000$

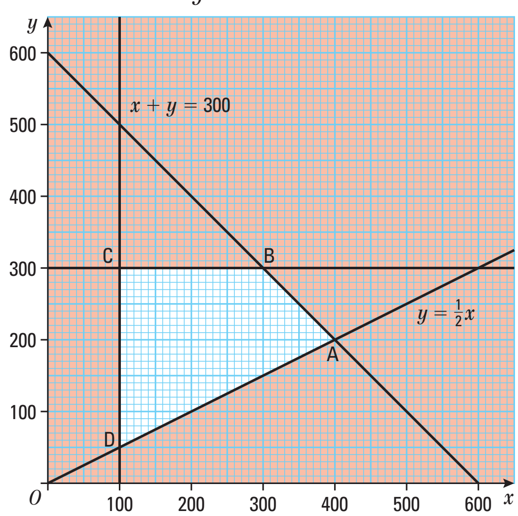


c $C = 20x + 30y$

d £12 000 **e** £9000

3 a $x \geqslant 100$, $y \leqslant 300$, $x + y \leqslant 600$, $y \geqslant \frac{1}{2}x$

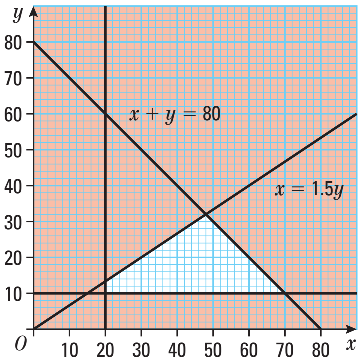
b $C = 100x + 160y$



d min = £18 000 at $x = 100$, $y = 50$,
max = £78 000 at $x = 300$, $y = 300$

4 **a** $x + y \leqslant 80$, $x \geqslant 1.5y$, $x \geqslant 20$, $y \geqslant 10$

b $R = 300x + 400y$

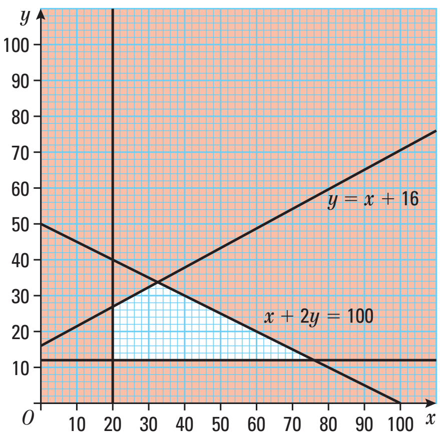


c R max = 27 200 at (48,32)

5 **a** **i** Total prize fund = $10x + 20y$, so $10x + 20y \leqslant 1000$

ii $x \geqslant 20$, $y \geqslant 12$ $y \leqslant x + 16$

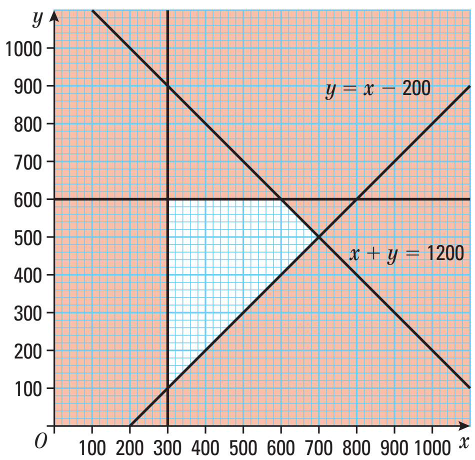
b



c Max value of $x + y$ is 88

6 **a** $x + y \leqslant 1200$, $x \geqslant 300$, $y \leqslant 600$, $y \geqslant x - 200$

b

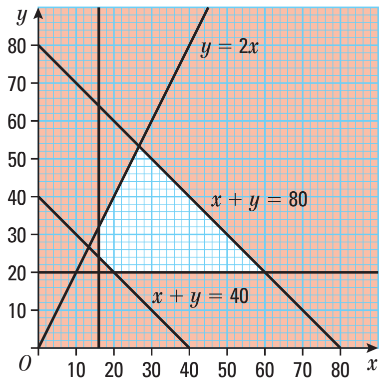


c $P = 20x + 10y$

d P max = 19 000

7 **a** $x \geqslant 16$ $y \geqslant 20$

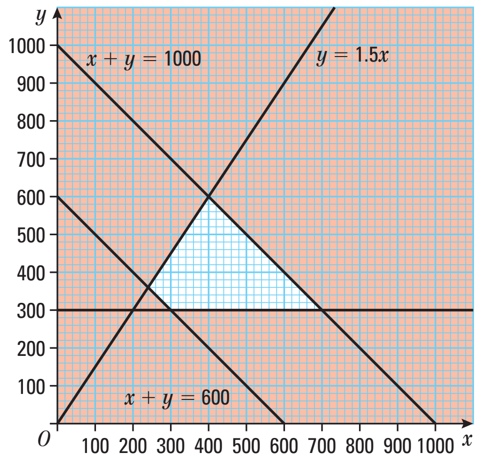
b $x + y \geqslant 40$ $x + y \leqslant 80$ $y \geqslant 2x$



c Maximum and minimum values of $150x + 250y$ are 17 300 and 8000

8 **a** $x \geqslant 200$, $y \geqslant 300$, $x + y \geqslant 600$,
$x + y \leqslant 1000$, $y \leqslant \frac{3}{2}x$

b



c £6900

A10.1 Gradients of graphs

Before you start

You should be able to:

- find the gradient of the line joining two points.

Why do this?

Aircraft engineers need to know about accelerations so that aircraft can be designed properly.

Objectives

- You can find an estimate for the gradient of a curve at any point by drawing a tangent to the curve.
- You can interpret the gradient of a curve as the rate of change of a quantity.

Get Ready

1 What is the gradient of the line segments which join these points?

 a A (2, 3) and B (4, 12)

 b C (4, 10) and D (6, 6)

 c E (−3, 3) and F (−1, 6)?

2 What does the phrase 'a tangent to a circle' mean?

Key Points

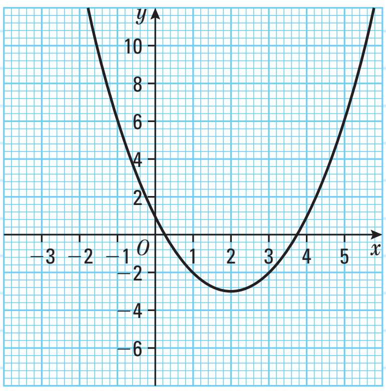
- The tangent at a point P on a graph is the straight line which just touches the graph at the point P.
- The gradient at a point on a graph is the gradient of the tangent to the graph at that point.
- The gradient of the tangent can be found in the same way as the gradient of any straight line.
- For a distance-time graph, the gradient is equal to the speed.
- For a speed-time graph or velocity-time graph, the gradient is equal to the acceleration.
- The acceleration of an object is equal to its rate of change of velocity.

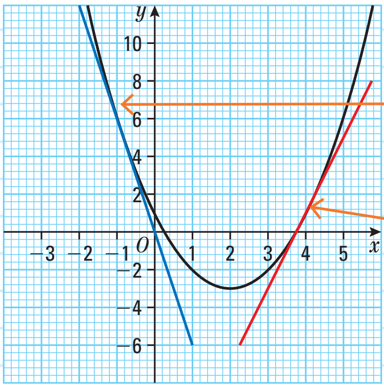
Example 1

Here is the graph of $y = x^2 - 4x + 1$

Work out an estimate for the gradient of the curve at:

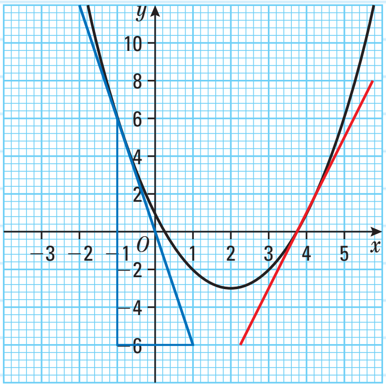
a $x = -1$ **b** $x = 4$





Draw the tangent to the curve at $x = -1$ (blue line)

Draw the tangent to the curve at $x = 4$ (red line)



Draw a triangle with the tangent as hypotenuse and work out $\frac{\text{Difference in } y \text{ values}}{\text{Difference in } x \text{ values}}$

− sign, because the tangent is downwards at the point where $x = -1$

From the diagram, at $x = -1$, gradient of the tangent $= \frac{6 - -6}{-1 - 1} = -6$,

at $x = 4$, gradient of the tangent $= \frac{5 - -3}{5 - 3} = 4$

Note that these are estimates as they depend on drawn tangents

Example 2

Rates of change

The average rate of change of a quantity is: $\frac{\text{Change in quantity}}{\text{time}}$

The water level in a tank was 40 cm at 01:00 and 90 cm at 05:00.

a Work out the average rate of change in the water level.

$\frac{90 - 40}{5 - 1} = 12.5$ cm per hour

$\frac{\text{Change in level}}{\text{time}}$

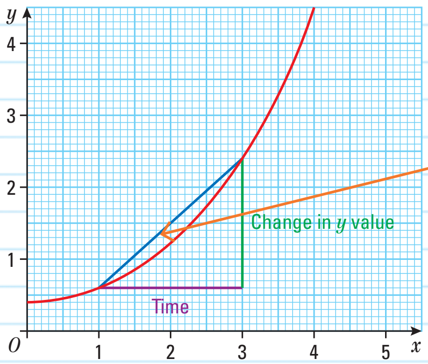
Between 05:00 and 10:00 the water level fell from 90 cm to 30 cm.

b Work out the average rate of change in the water level.

$\frac{\text{Change in level}}{\text{time}} = \frac{30 - 90}{10 - 5} = -12$ cm per hour

− sign because the water level decreases through the time interval

For a graph of a quantity which varies with time, the average rate of change of the quantity is equal to the gradient of the line segment joining the two points over which the quantity changes.

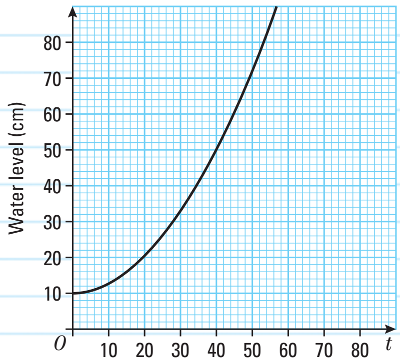


Average Rate of change
$= $ Gradient of line segment
$= \dfrac{\text{Change in quantity}}{\text{time}}$

$\dfrac{2.4 - 0.6}{3 - 1} = 0.9$ units per time

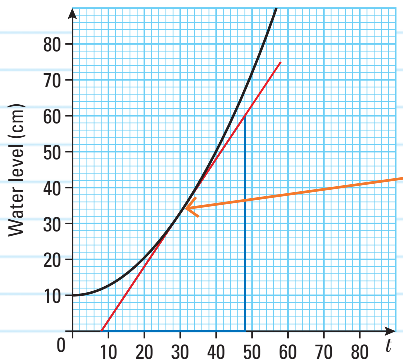
The instantaneous rate of change of a quantity at a given time is equal to the gradient of the tangent to the graph of how the quantity changes at that time.

Example 3 Here is the graph of how the water level changes in a water tank.



The instantaneous rate of change of a quantity at a given time, is equal to the gradient of the tangent to the graph of how the quantity changes at that time.

Find an estimate for the rate of change of water level at $t = 30$.



Draw the tangent to the curve at $t = 30$. Draw a suitable triangle and work out the gradient.

Rate of change of water level (at $t = 30$) $= \dfrac{60 - 0}{48 - 8} = 1.5$ cm per sec

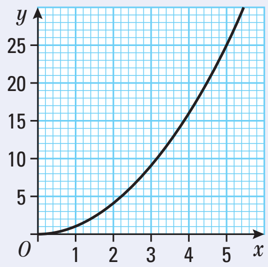
For distance-time graphs, the gradient of the graph at any point gives the speed at that instant.
For velocity-time graphs, the gradient of the graph at any point gives the acceleration at that instant.

Exercise 10A

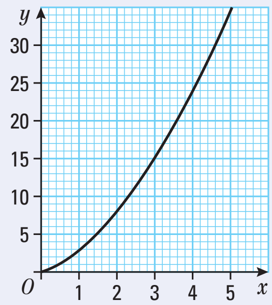
B

1 Find the gradients of the following curves at the values of x given.

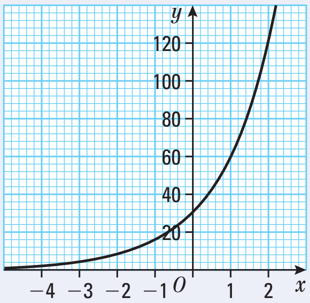
a At $x = 3$



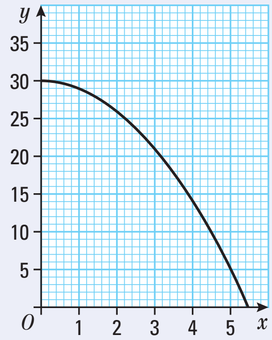
b At $x = 2$



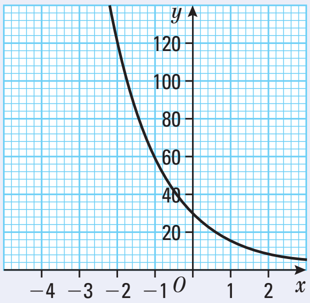
c At $x = -1$



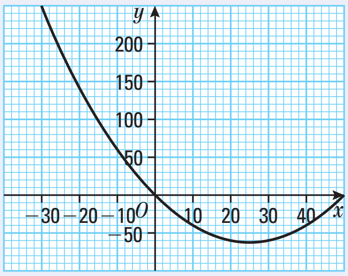
d At $x = 2$



e At $x = -1$



f At **i** $x = -20$ and **ii** $x = 30$



2 Draw the curve with equation $y = x^2$ for values of x from -3 to 3.
Calculate an estimate of the gradient of the curve at these points:

a $(-2,4)$ **b** $(1,1)$

A

3 **a** Copy and complete the table of values for the curve with equation $y = x^2 + 4x$ for values of x from 0 to 5.

x	0	1	2	3	4	5
y		5				45

b Draw the graph.

c Calculate an estimate for the gradient of the graph at $x = 2$.

A

4 **a** Copy and complete the table of values for the curve with equation $y = 8x - x^2$.

x	0	1	2	3	4	5	6
y		7				15	

b Draw the graph.

c Calculate an estimate for the gradient of the graph:

i at $x = 2$ **ii** at $x = 4$

5 **a** Copy and complete the table of values for the curve with equation $y = x^2 - 4x - 5$.

x	-3	-2	-1	0	1	2	3
y		7				-9	

b Draw the graph.

c **i** Calculate an estimate for the gradient of the graph at $x = 1$.

ii Calculate an estimate for the gradient of the graph at $x = -2$.

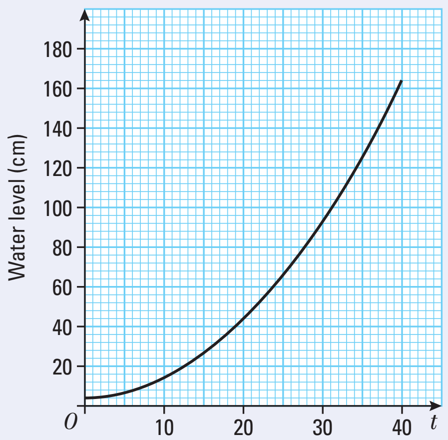
6 **a** Copy and complete the table of values for the curve with equation $y = \frac{24}{x}$

x	1	2	3	4	5	6
y	24				4.8	

b Draw the graph.

c Calculate an estimate for the gradient of the graph at $x = 4$.

7 The graph shows the water level in a tank.

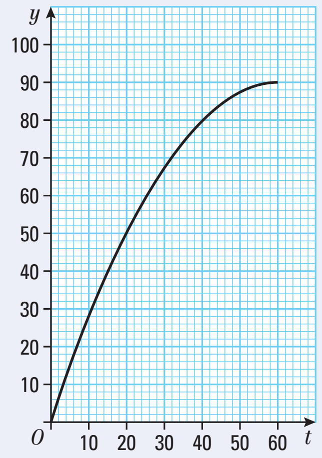


a Work out an estimate for the rate at which the water is rising when $t = 15$

b Work out the average rate of rise of the water level between $t = 10$ and $t = 30$

A

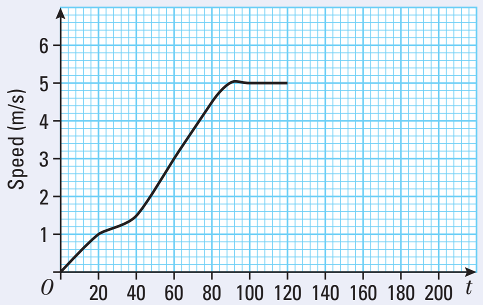
8 The graph shows the distance, y m, that a car has travelled during t seconds.



a Calculate an estimate of the speed of the car at $t = 20$.

b Calculate the average speed of the car between $t = 10$ and $t = 50$.

9 The graph shows the velocity of a train for the first two minutes after it had left a station.



Calculate an estimate of the acceleration of the train after:

a 20 seconds **b** 80 seconds.

For the 3rd minute the train reduces speed at a constant rate until it comes to rest.

Draw the velocity-time graph for the first 3 minutes of the train's journey and find the deceleration of the train.

10 The graph shows the distance that a car has travelled in its first minute measured from a point X on a road.

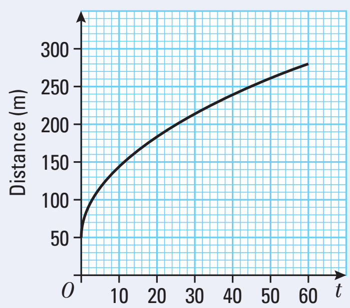
a Calculate an estimate of the speed of the car at $t = 25$.

A van travelling in the same direction along the same road has the distance d metres, it travels in t seconds from the point X, given by the equation $d = 5t$.

b How far ahead of the van was the car initially?

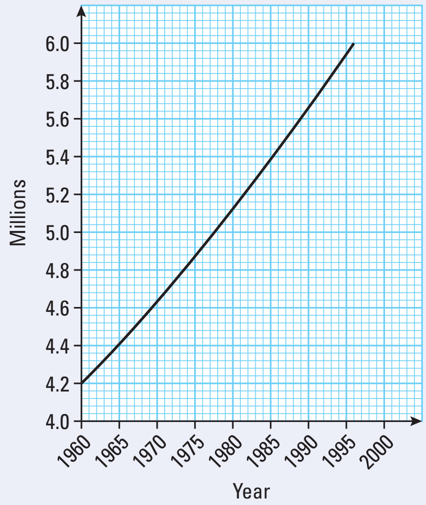
c Describe fully the motion of the van.

d Use the graph to find an estimate of the value of t when the van catches up with the car.



A

11 The graph shows how the population of a city changed with time.



Calculate an estimate of the rate of change of the population at the start of 1980.

Review

- The gradient of a curve at a point is the same as the gradient of the tangent to the curve at that point.
- The gradient of a distance-time graph at a time t is equal to the velocity at time t.
- The gradient of a velocity-time graph at a time t is equal to the acceleration at time t.

Answers

Chapter 10

A10.1 Get Ready answers

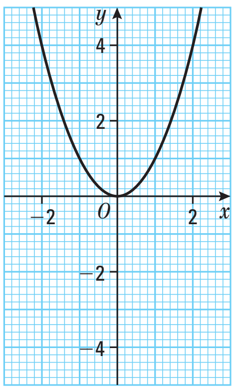
1 **a** 4.5 **b** −2 **c** 1.5

2 A tangent to a circle is a straight line which touches the circle. (More technically, it intersects the circle at two coincident points)

Exercise 10A

1 **a** 6 **b** 6 **c** 10
d −4 **e** −10 **f** **i** −9, **ii** 1

2

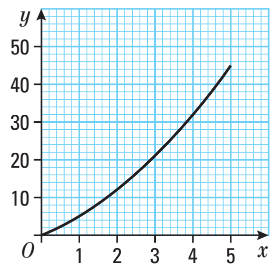


a −4 **b** 2

3 **a**

x	0	1	2	3	4	5
y	0	5	12	21	32	45

b

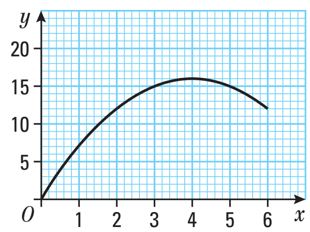


c 8

4 **a**

x	0	1	2	3	4	5	6
y	0	7	12	15	16	15	12

b

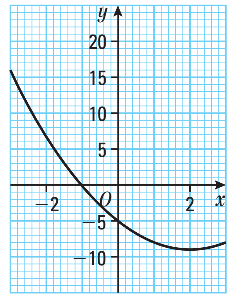


c **i** 4 **ii** 0

5 **a**

x	−3	−2	−1	0	1	2	3
y	16	7	0	−5	−8	−9	−8

b

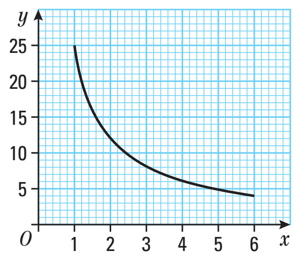


c **i** −2 **ii** −8

6 **a**

x	1	2	3	4	5	6
y	24	12	8	6	4.8	4

b

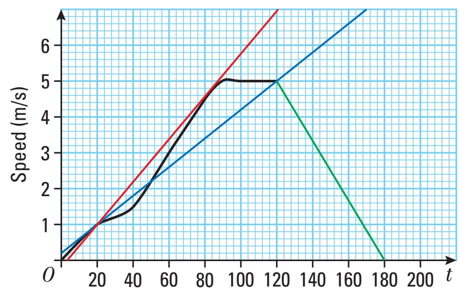


c −1.5

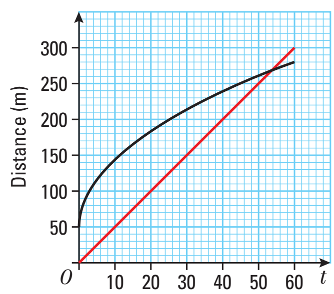
7 **a** 3 cm/sec **b** 4 cm/sec

8 **a** 2 m/s **b** 1.5 m/s

9 **a** 0.04 m/s/s **b** 0.06 m/s/s
c Deceleration is 5 ÷ 60 = 0.083 m/s/s



10 **a** 3 m/s **b** 50 m
c Moving at a constant speed of 5 m/s.
d After 54 seconds



11 0.6 ÷ 10 = 0.06 million per year.

A12.1 Time series graphs

Before you start

You should be able to:

- draw, label and scale axes
- plot points on a coordinate grid.

Why do this?

You might want to show how sales figures are changing over a period of time.

Objectives

- You can represent data using a time series graph.
- You can identify seasonality and trends in time series.

Get Ready

1. Write down a list of six numbers which are increasing.
2. Write down a list of six numbers which are decreasing.
3. Write down a list of six numbers which are neither increasing nor decreasing.

Key Points

- A graph showing how a given value changes over time is called a time series graph.
- You can use a time series graph to identify whether there is any seasonal variation in the data – for example, if there is a peak or a trough at the same time each year.
- A time series can help you to identify whether there is any trend in the data.

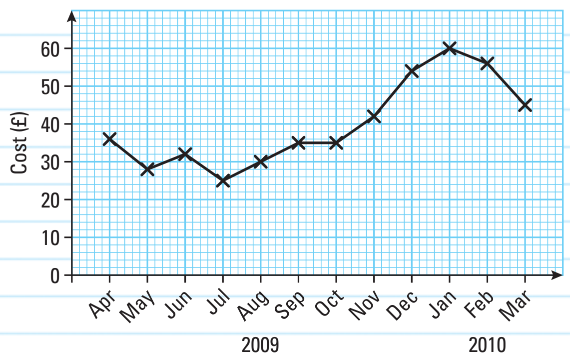
Example 1

The table below gives information about the cost of the gas Angela used each month between April 2009 and March 2010.

Month	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Jan	Feb	Mar
Cost (£)	36	28	32	25	30	35	35	42	54	60	56	45

a Draw a time series graph to show this information.
b In which month did Angela spend most on gas?
c Explain how the cost of gas changes over the year.

a



Plot the points on the grid.

Join the points with straight lines.

Find when the highest value occurs.

b January

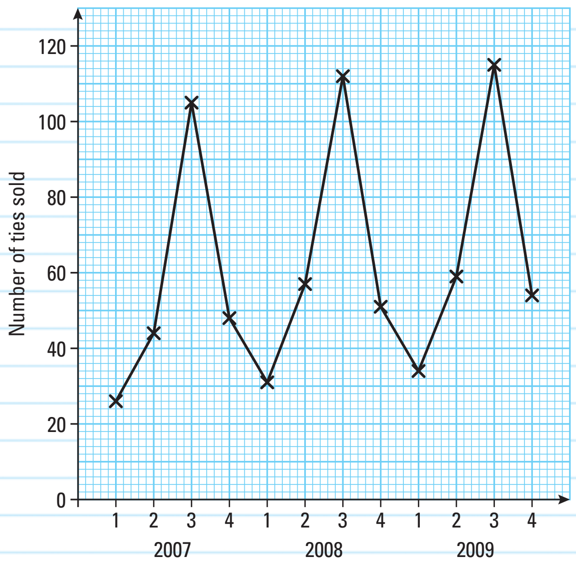
c The cost of gas decreases during the first half of the year then increases in the second half of the year.

Example 2 The table shows the number of ties sold in a school shop in each quarter of three successive years.

Year	Quarter			
	1	2	3	4
2007	26	44	105	48
2008	31	57	112	51
2009	34	59	115	54

a Plot the time series graph.
b In which quarter is the sale of ties highest?
c Describe the trend in the number of ties sold.

a



Find the quarter in which most ties are sold each year.

Note that there is a seasonal variation in the number of ties sold. The greatest number of ties sold is always in Quarter 3.

Although the number of ties sold varies greatly from quarter to quarter the trend in the number of ties sold is upwards.

b *Quarter 3*

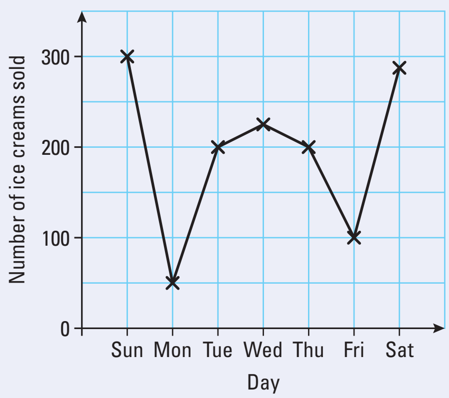
c *The number of ties sold is increasing over time.*

Exercise 12A

F A02 A03

1 The graph shows the number of ice creams sold each day during one week.

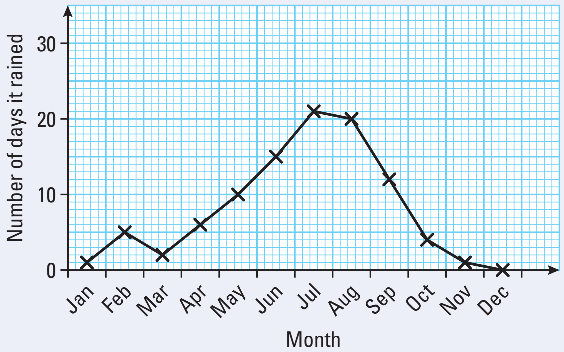
How many more ice creams were sold on Tuesday than on Monday?



(June 2006)

A02 E

2 The graph shows information about the rainfall in Kathmandu.
It shows the number of days it rained each month.



a Write down the number of days it rained in April.

b In which month did it rain most?

One month it rained on exactly 12 days.

c Which month? *(March 2008, adapted)*

A03 D

3 The table shows the number of cars sold by a garage each month from July to December.

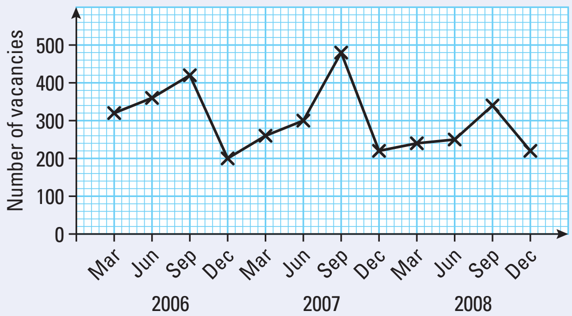
July	August	September	October	November	December
28	26	25	21	22	17

a Draw a time series graph to show this information.

b Describe the trend in the number of cars sold at this garage.

A03 C

4 This graph shows the number of job vacancies in a town from 2006 – 2008.



a Describe the seasonal variation in the number of job vacancies.

b Describe the trend in the number of job vacancies over the three years.

A12.2 Moving averages

Before you start

You should be able to:

- work out the mean of a set of numbers
- draw a line of best fit.

Why do this?

The number of cars sold by a garage might vary considerably according to the time of year. Moving averages may be used to show whether the general trend in number of cars sold is up or down.

Objectives

- You can calculate moving averages.
- You can use moving averages to identify trends.

Get Ready

Work out the mean of:

1 46, 51, 44

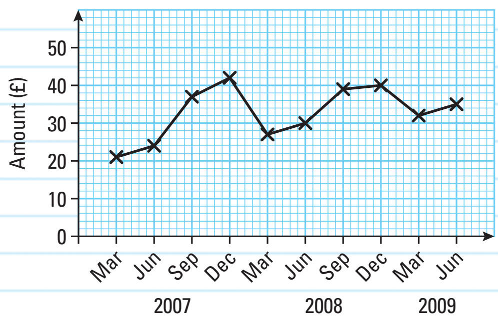
2 £680, £820, £745, £813

Key Points

- To find the three-point moving averages for a time series , work out the average of the first, second and third values, then the average of the second, third and fourth values and so on.
- To find four-point moving averages, we use four values at a time, for five-point moving averages, five values and so on.
- A moving average gives a value which changes over time.
- Moving averages are used to smooth out variation in a set of values. For example, they can be used to smooth out seasonal variation.
- Plotting moving averages on a time series graph helps you to identify any general trend in the data.
- A moving average is plotted at the midpoint of the values used to generate it.

Example 3 The table and graph show the amounts on 10 of Simon's electricity bills.

Month	Mar 2007	June 2007	Sept 2007	Dec 2007	Mar 2008	June 2008	Sept 2008	Dec 2008	Mar 2009	June 2009
Amount (£)	21	24	37	42	27	30	39	40	32	35



There are four values for each year so use four-point moving averages.

a Calculate suitable moving averages for the data.

b Plot the moving averages on the same graph as the original data.

c Comment on the trend in Simon's electricity bills.

a First moving average $M_1 = \frac{21 + 24 + 37 + 42}{4} = 31$

Firstly, work out the average of the first four values in the table.

Second moving average $M_2 = \frac{24 + 37 + 42 + 27}{4} = 32.5$

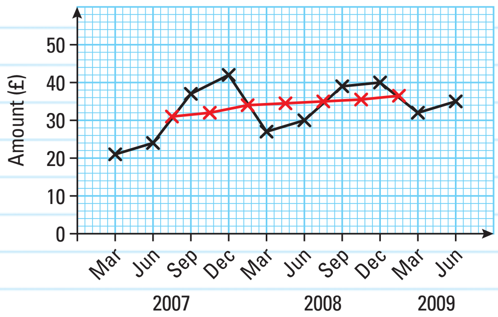
Similarly $M_3 = \frac{37 + 42 + 27 + 30}{4} = 34$ and so on.

Work out the second moving average by moving up the list one place.

The four point moving averages for the whole data set are:

M_1	M_2	M_3	M_4	M_5	M_6	M_7
31	32.5	34	34.5	35	35.25	36.5

b



Plot each moving average at the midpoint of the values used to generate it. Join the moving averages with straight lines.

c There is an upward trend in Simon's electricity bills over the period March 2007 to June 2009.

Use the moving averages on the graph to identify the trend.

Example 4

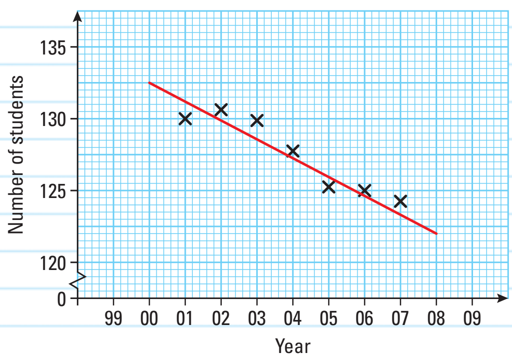
The number of students in year 7 at Colbury School at the beginning of the school year for the years 1999 – 2009 were:

Year	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Students	126	129	128	135	132	129	125	118	122	131	125

a Work out the five-point moving averages for this data.
b Plot the moving averages and draw a trend line on your graph.
c Comment on how the number of pupils in year 7 has changed over the years 1999 to 2009.

a The five point moving averages are 130, 130.6, 129.8, 127.8, 125.2, 125, 124.2

b



Plot each of the moving averages at the interval midpoint.

Draw a trend line by drawing a line of best fit for these points.

Interpret the trend.

c The number of pupils in year 7 has decreased over the period 1999 to 2000.

Exercise 12B

B

AO1

1 The table shows the number of computer games sold in a supermarket each month from January to June.

Jan	Feb	Mar	Apr	May	June
147	161	238	135	167	250

Work out the three-month moving averages for this information. *(June 2004)*

AO1

2 The table shows the number of orders received each month by a small company.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Number of orders received	23	31	15	11	19	16	20	13

Work out the first two four-month moving averages for this data. *(June 2003)*

AO3

3 A shop sells DVD players.

The table shows the number of DVD players sold in every three-month period from January 2003 to June 2004.

Year	Months	Number of DVD players sold
2003	Jan – Mar	58
	Apr – Jun	64
	Jul – Sep	86
	Oct – Dec	104
2004	Jan – Mar	65
	Apr – Jun	70

a Calculate the set of four-point moving averages for this data.
b What do your moving averages in part **a** tell you about the trend in the sale of DVD players?

(March 2005)

AO3

4 Jasmine sells soft drinks. She recorded the number of soft drinks she sold from July to December.

The table shows this information.

July	August	September	October	November	December
340	352	336	272	256	264

a Work out the four-month moving averages for this information.
b What do your moving averages tell you about the sales of soft drinks from July to December?

(Summer 2007, adapted)

A02 B

5 Joe owns a small shop.
The table shows his sales, in £, in the eight 3-month periods for the last two years.

		3-month period	**Sales in £**
Year 1	1	January to March	3420
	2	April to June	3370
	3	July to September	3750
	4	October to December	4020
Year 2	1	January to March	3940
	2	April to June	3810
	3	July to September	4230
	4	October to December	4560

The first four-point moving averages have been worked out.

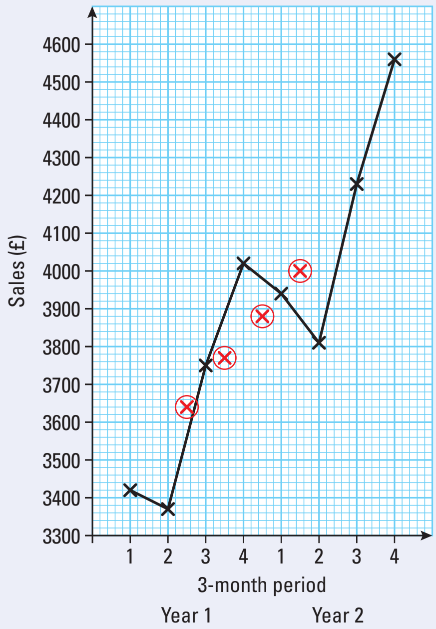
a Work out the fifth four-point moving average.

£3640, £3770, £3880, £4000, £......

The time series graph shows Joe's sales for the last two years.
The first four four-point moving averages have been plotted on the grid.

b Plot the fifth four-point moving average.

c Draw a trend line for the data.



(Novemeber 2007)

A03

6

Month	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**
Number of Televisions	1240	1270	1330	1300	1330	x

The table shows the number of televisions sold in a shop in the first five months of 2006.

a Work out the first 3-month moving average for the information in the table.

The fourth 3-month moving average of the number of televisions sold in 2006 is 1350.
The number of televisions sold in the shop in June was x.

b Work out the value of x.

(Novemeber 2007)

B A03

7 The table shows the number of pupils at a dance class each week for 10 weeks.
The table also shows seven of the three-point moving averages.

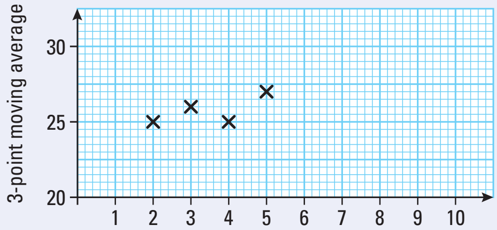
Week	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Number of pupils	23	25	27	26	22	33	23	25	30	29
3-point moving average		25	26	25	27	26	27	26		

a Work out the missing three-point moving average.

b Copy the grid and plot the three-point moving averages from your table.
The first four have been plotted for you.

c On the grid, draw a trend line.

d Comment on the trend shown by your graph.



(Summer 2008)

A02 A03

8 The table shows the number of strawberry plants sold by a garden centre over four days.

	Morning	**Afternoon**	**Evening**
Monday	81	99	78
Tuesday	93	93	54
Wednesday	51	54	18
Thursday	12	33	21

a Calculate the values of a suitable moving average.
b Plot the original data and the moving averages on the same graph.
c Comment on your graph.

Review

- A graph showing how a given value changes over time is called a **time series graph**.
- You can use a time series graph to identify whether there is any **seasonal variation** in the data – for example, whether there is any variation in sales figures at different times of the year.
- A time series can help you to identify whether there is any **trend** in the data.
- To find the three-point **moving averages** for a time series , work out the average of the first, second and third values, then the average of the second, third and fourth values, and so on.
- To find four-point moving averages, we use four values at a time, for five point moving averages, five values, and so on.
- A moving average gives a value which changes over time.
- Moving averages are used to smooth out variation in a set of values. For example, they can be used to smooth out seasonal variation.
- Plotting moving averages on a time series graph helps you to identify any general trend in the data.
- A moving average is plotted at the midpoint of the values used to generate it.
- To draw a graph of the moving averages, plot the moving averages and join the points with straight lines.
- A **trend line** is obtained by drawing a line of best fit for the moving average points.

Answers

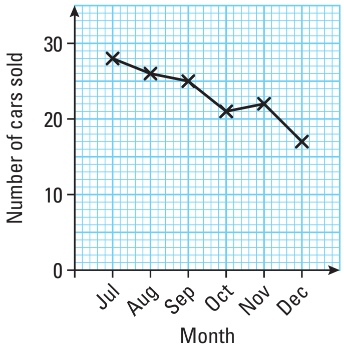
Chapter 12

A12.1 Get Ready answers

1 Answers will vary.
2 Answers will vary.
3 Answers will vary.

Exercise 12A

1 150
2 a 6 b July c September
3 a



b The number of cars sold is decreasing over time.
4 a The number of job vacancies is greatest in September each year and least in December each year.
b The number of job vacancies decreased over the three years.

Exercise 12B

1 182, 178, 180, 184
2 20, 19
3 a 78, 79.75, 81.25
b There is an upward trend in the sale of DVD players.
4 a 325, 304, 282
b The trend is that sales are falling.
5 a 4135
b point (6.5, 4135) plotted
c line of best fit drawn for moving averages.
6 a 1280 b 1420
7 a 28
b points (6, 26), (7, 27), (8, 26), (9, 28) plotted
c Trend line drawn
d The number of pupils at a dance class increases over the 10 weeks.
8 a 86, 90, 88, 80, 66, 53, 41, 28, 21, 22
b Original data and moving averages plotted
c General trend is a decrease in the number of strawberry plants sold.

A13.1 Risk

Before you start

You should be able to:

- calculate an estimate for the total number of successes in a series of identical trials, when the probability of success on any trial is constant and known
- construct probability tree diagrams and use them to work out the probability of compound events.

Why do this?

The risk of an event is related to its probability and to its impact (usually measured financially) so risk calculations are carried out every day by insurance companies.

Objectives

- You will gain an understanding of risk.
- You will be able to carry out calculations involving the concept of risk.

Get Ready

1. Jim rolls a fair dice 200 times. Work out an estimate for the number of times he should get the number 1.
2. Jim throws a fair coin and rolls a fair dice. Use a probability tree diagram to work out the probability he gets either a head or a score greater than 4, but not both.

Key Points

- The risk of an event is the probability that it will happen.
- Risks are often presented as relative frequencies such as 1 in 100.
- An estimate of the cost of an event can be obtained by multiplying the probability of the event by the actual cost if the event did happen.

Example 1

The probability of a washing machine flooding a kitchen in any one year is 0.001.
An insurance company pays out £800 for each flooded kitchen.
The insurance company insures 20 000 washing machines.
Work out an estimate of the amount of money that the insurance company must pay out next year.

Estimated number of flooded kitchens next year $= 0.001 \times 20\,000 = 20$
Estimated total amount of money to be paid out $= 20 \times £800 = £16\,000$

Example 2

The table gives information about the number of trains that ran and the number of those trains that were late during one month.

Time Period	Number of trains that ran	Number of trains that were late
06 : 00 – 07 : 00	347	34
07 : 00 – 08 : 00	428	43
08 : 00 – 09 : 00	517	40
09 : 00 – 10 : 00	326	28

a Compare between the four time periods, the risks of having a late train.

Transport engineers estimate that the cost to the train company of a late train is £3000.
The company plans to run 450 trains next month between 06 : 00 and 07 : 00.

b Estimate the cost of late trains to the train company if no improvements are made to lateness.

a 06 : 00 – 07 : 00 Prob of being late $= \frac{34}{347} = 0.0980$

The risk of any one train being late in this period is 0.0980, correct to 3 significant figures.

07 : 00 – 08 : 00 Prob of being late $= \frac{43}{428} = 0.100$

08 : 00 – 09 : 00 Prob of being late $= \frac{40}{517} = 0.0774$

09 : 00 – 10 : 00 Prob of being late $= \frac{28}{326} = 0.0859$

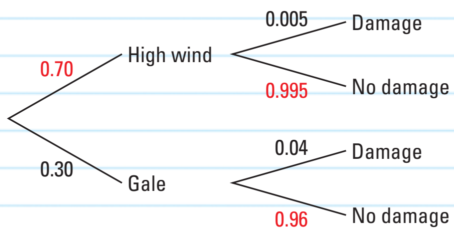
One way to compare is to use the figures to calculate probabilities of being late.

In order of reliability (most reliable first), the periods are:
08 : 00 – 09 : 00, 09 : 00 – 10 : 00, 06 : 00 – 07 : 00, 07 : 00 – 08 : 00

b An estimate of the number of late trains $= 450 \times 0.0980 = 44.1$
An estimate of the cost $= 44.1 \times £3000 = £132\,300$

Example 3

A company generates electricity from an offshore site with wind turbines.
If a high wind becomes a gale the probability of damage to the wind turbines increases.
The probability of damage in a gale is 0.04.
The probability of damage in a high wind is 0.005.
The probability that a high wind becomes a gale is 0.3.
This site has 50 high wind days each year.
Work out an estimate for the number of times it will be damaged in a period of 10 years.



The probability tree has been used to show the structure of the problem, so that the probability of damage on any high wind day can be found.
The red figures have been calculated by subtraction from 1.

Probability of damage on one day $= 0.7 \times 0.005 + 0.3 \times 0.04$
$= 0.0155$

Number of times it will be damaged $= 0.0155 \times 50 \times 10 = 7.75$

Either:
high wind and damage
or
gale and damage

Exercise 13A

C

1 Last year, a manufacturer sold 18 500 dishwashers. Of these, 121 broke down.

a Work out an estimate of the probability of a dishwasher breaking down.

This year, the manufacturer will sell 16 850 washing machines.

b Work out an estimate of the number of washing machines that will break down.

C

2 The table gives information about the number of repairs to electrical appliances made by a company.

Type of Appliance	Number made	Number of repairs
Washing Machine	12 800	198
Dishwasher	17 484	321
Dryer	13 724	216
Fridge	9515	125

Compare the risk of breakdown for each type of appliance.

3 An insurance company insures computers against breakdown. Last year, out of a total of 16 700, 84 computers broke down.

a Work out the probability of a computer breaking down.

The cost of repairing or replacing a computer is £528.

Next year, the number of computers insured will be 18 250.
Work out an estimate of the price that the insurance company should charge for it to break even.

4 A supermarket company owns 4000 freezers. The probability of a freezer breaking down in a year is 0.005. When the freezer breaks down the supermarket estimates the cost of repair and replacement as £800.

Work out an estimate for the cost to the supermarket company of repairs and replacements due to its freezers breaking down.

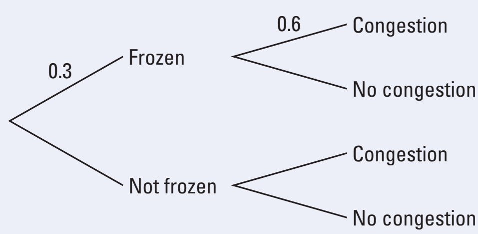
5 The table gives information about the number of trains that ran and the number of those trains that were late during one month.

Time Period	Number of trains that ran	Number of trains that were late
16 : 00 – 17 : 00	285	30
17 : 00 – 18 : 00	401	55
18 : 00 – 19 : 00	480	40
19 : 00 – 20 : 00	303	31

a Compare between the four time periods, the risks of having a late train.
Transport engineers estimate that the cost to the train company of a late train is £3400.
The company plans to run 25 more trains during each time period next month.

b Work out an estimate of the cost of late trains to the train company next month if no improvements are made to lateness.

6 A town council is working out an estimate of costs to the town due to frozen roads.
The probability tree diagram gives some information about the probabilities of frozen roads and of congestion in the town during 60 days in winter.



The town council estimates that the probability of congestion is 0.1, if there are no frozen roads.

a Copy and complete the tree diagram.

b Work out the probability that on any given day in winter, there will be congestion.

The town council thinks that if there is congestion on any day the cost to the town is £1200.

c Work out an estimate of the cost to the town during these 60 days.

C

7 Last year there were 7685 landings at an airport. Of these landings there were 16 in which the aircraft suffered damage to tyres.

a Calculate an estimate of the risk of damage to an aircraft on landing at this airport.

A damaged aircraft tyre costs £650.

Next year the airport estimates that there will be 8100 landings.

b Work out an estimate of the total cost of damage to tyres at this airport.

B

8 The table gives information about the reliability of different makes of washing machines used last year.

Make	Number sold	Cost (£)	Number of breakdowns
Kandoo	3450	399	38
Black Diamond	4970	329	52
Illustrious	6500	259	79
Dekko	7680	199	125

a Use the 'number sold' column and the 'number of breakdowns' column to work out an estimate of the risk of each make of washing machine breaking down.

b Use the columns to work out which make is the best value.

9 A central heating company made a comparison between those households which had had their boiler serviced that year and those that had not. Information about this is given in the table.

	Number serviced	Number not serviced
Number of breakdowns	23	56
Number not breaking down	287	320

Jim has a boiler. The cost of a service is £50. The average cost of a repair if the boiler breaks down is £145.

On average is it cheaper for Jim to have the service?

A

10 Around a coastline, 60% of towns have flood defences. If a town has flood defences then the probability that there is flooding is 0.01 in any year. If a town does not have flood defences then the probability of flooding is 0.02 in any year.

a Work out the probability of flooding in a town in any year.

The cost of dealing with a flood in a town is £5 million.

There are 20 towns along the coastline.

b Work out an estimate of the cost of dealing with floods over the next 10 years.

A*

11 If Jim's train gets in on time he can then catch a bus costing £2. If the train is late he must then catch a taxi costing £10.

The probability that the train will be late is 0.1.

Work out an estimate of how much Jim will have to pay on average.

12 Mattie could spend 20 min on homework or watch the TV instead. The probability that her teacher will ask for the homework is 0.7. If she finds that Mattie has not done her homework then she will give a three-quarters of an hour detention.

What should Mattie do?

A*

13 If I get my central heating serviced then the probability that it will fail in the next year is 0.04. If I do not get it serviced the probability that it will fail in the next year is 0.1.

The cost of a service is £50. The likely cost of a repair if it fails is £230.

What are the financial implications?

14 Jim has £10 000 to invest. He considers investing in one or both of two investments:
Investment 1: The Cautious Investor fund: Percentage yield = 15%
Investment 2: The High Stakes investor fund: Percentage yield = 45%

Both investments involve risk.

For the Cautious Investor fund the risk of losing half the initial investment is 1%.
For the High Stakes investor fund the risk of losing half the initial investment is 28%.

Jim considers 3 different investment plans.
A Invest all the money in the Cautious Investor fund.
B Invest all the money in the High Stakes investor fund.
C Invest half in the Cautious Investor fund and half in the High Stakes investor fund.

Compare the 3 different investment plans.

Review

- Given that the cost of a breakdown is £C and the probability of a breakdown is p then an estimate of the risk cost of the breakdown is £pC.

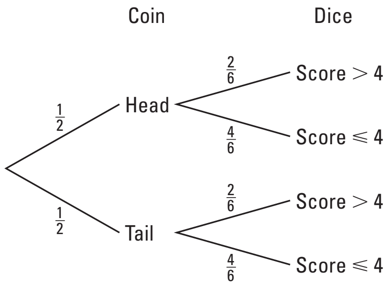
Answers

Chapter 13

A13.1 Get Ready answers

1 33.3

2



Probability $= \frac{1}{2}$:

Exercise 13A

1 **a** $\frac{121}{18500} = 0.006\,54$

b $16850 \times 0.006\,54 = 110$

2

Washing Machine	0.0155
Dishwasher	0.0184
Dryer	0.0157
Fridge	0.0131

In order with the least risky first:
Fridge, Washing machine, Dryer, Dishwasher

3 **a** 0.005 03

b $18\,250 \times 0.005\,03 \times £528 = £48\,469$,
$£48\,469 \div 18\,250 = £2.66$

4 $4000 \times 0.005 \times 800 = £16\,000$

5 **a** 16 : 00 – 17 : 00 Prob of being late $= \frac{30}{285} = 0.105$

17 : 00 – 18 : 00 Prob of being late $= \frac{55}{401} = 0.\,137$

18 : 00 – 19 : 00 Prob of being late $= \frac{40}{480} = 0.\,0833$

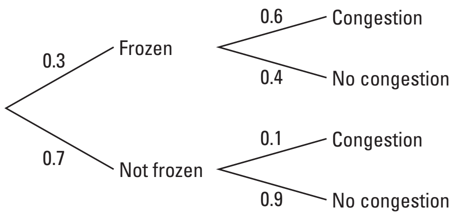
19 : 00 – 20 : 00 Prob of being late $= \frac{31}{303} = 0.\,102.$

In order of reliability the periods are:
18 : 00 – 19 : 00, 19 : 00 – 20 : 00,
16 : 00 – 17 : 00, 17 : 00 – 18 : 00

b An estimate of the number of late trains $=$
$310 \times 0.105 + 426 \times 0.137 + 505 \times 0.0833 +$
$328 \times 0.102 = 166$

An estimate of the cost $= 166 \times £3400 = £564\,400$

6 **a**



b $0.3 \times 0.6 + 0.7 \times 0.1 = 0.25$

c $0.25 \times 60 = 15$, $15 \times £1200 = £18\,000$

7 **a** $\frac{16}{7685} = 0.002\,082$

b $8100 \times 0.002\,082 \times £650 = £10\,962$

8 **a**

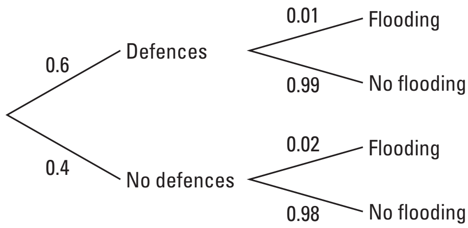
Make	Probability
Kandoo	0.0110
Black Diamond	0.0105
Illustrious	0.0122
Dekko	0.0163

b Kandoo $£399 \times 0.0110 = £4.39$,
Black diamond $£329 \times 0.0105 = £3.45$,
Illustrious $£259 \times 0.0122 = £3.16$
Dekko $= £199 \times 0.0163 = £3.24$

So Illustrious is the best value.

9 Prob of breaking down if serviced $= \frac{23}{310}$
Estimated cost $= \frac{23}{310} \times 145 + 50 = £60.76$
Prob of breaking down if not serviced $= \frac{56}{376}$
Estimated cost $= \frac{56}{376} \times 145 = £21.60$
From a cost point of view he should not have the service.

10



a $0.6 \times 0.01 + 0.4 \times 0.02 = 0.014$

b $20 \times 0.014 \times 10 \times £5$ million $= £14$ million

11 $0.9 \times £2 + 0.1 \times £10 = £2.80$

12 An estimate for the number of minutes of detention is $0.7 \times 45 = 31.5$

So Mattie should spend 20 minutes on her homework.

13 Serviced $0.04 \times £230 + £50 = £59.20$
Not serviced $0.1 \times £230 = £23$
Better to not have it serviced.

14 A Value after one year: $£10\,000 \times 1.15 = £11\,500$.
Loss $= 0.01 \times £5000 = £50$
$£11\,500 - £50 = £11\,450$

B $£10\,000 \times 1.45 = £14\,500$.
Loss $= 0.28 \times £5000 = £1400$
$£14\,500 - £1400 = £13\,100$

C $£5000 \times 1.15 = £5750$. Loss $= 0.01 \times £2500 = £25$
$£5750 - £25 = £5725$
$£5000 \times 1.45 = £7250$.
Loss $= 0.28 \times £2500 = £700$
$£7250 - £700 = £6550$
Total $= £12\,275$
Plan B offers the possibility of a high yield. Even factoring in the possible loss it gives the highest yield.