A6.1 AER and compound interest

Before you start

- You should already know how to increase an amount by a given percentage.
- You will need to be able to use your calculator to find the nth root of a number.

Why do this?

Compound interest and the annual equivalent rate (AER) play an important role in everyday investments, especially those taking place over more than two or three years.

Objectives

- Be able to calculate the final amount and the interest on an investment.
- Be able to calculate the annual equivalent rate (AER) of an investment.

Get Ready

1 £6000 is invested at 4% p.a. Work out the value of the investment after one year.

2 Use a calculator to work out:

a 2^{10} **b** $6000 \times \left(1 + \frac{4}{100}\right)^5$

3 Use a calculator to work out $729^{\frac{1}{6}}$

Key Points

- Compound interest is interest paid on the amount and the interest already earned.

Example 1

Katie invests £3000 at 3.4% compound interest.
Work out the value of her investment after 2 years.

$$\text{Interest after 1 year} = \frac{3000 \times 3.4}{100} = £102$$

Value after 1 year = investment + interest after 1 year

$$\text{Value after 1 year} = 3000 + 102 = £3102$$

$$\text{Interest in year 2} = \frac{3102 \times 3.4}{100} = £105.47$$

Value after 2 years = value after 1 year + interest in year 2

$$\text{Value after 2 years} = 3102 + 105.47 = £3207.47$$

Exercise 6A

C

1 Jim invests £2000 at 3% p.a. compound interest for 2 years. Work out the final amount.

2 Jade invests £1500 at 3% compound interest for 10 years. Work out the final amount.

Key Points

- Compound interest can also be calculated using a formula.
- When £P is invested in an account paying r% compound interest per annum (p. a.), the value, £V, of the investment after n years is given by:

$$V = P\left(1 + \frac{r}{100}\right)^n$$

- When £P is invested in an account for n years to produce an investment of value £V, the annual equivalent rate of interest (AER) is given by:

$$\alpha = 100\left(\left(\frac{V}{P}\right)^{\frac{1}{n}} - 1\right) \text{ where } \left(\frac{V}{P}\right)^{\frac{1}{n}} = \sqrt[n]{\left(\frac{V}{P}\right)}$$

Example 2

Katie invests £3000 at 3.4% compound interest.
Work out the value of her investment after 5 years.

$$V = 3000 \times \left(1 + \frac{3.4}{100}\right)^5 = 3000 \times 1.034^5 = £3545.88$$

Substitute $P = 3000$, $r = 3.4$ and $n = 5$ into the compound interest formula.

Example 3

Josh invested £5000 in an account.
After 5 years the value of the account was £7000.
Work out the annual equivalent rate (AER) of the account.

$$\alpha = 100 \times \left(\left(\frac{7000}{5000}\right)^{\frac{1}{5}} - 1\right) = 100 \times \left(1.4^{\frac{1}{5}} - 1\right) = 6.96\%$$

Substitute $P = 5000$, $V = 7000$, and $n = 5$ into the formula

Example 4

Adam invested some money into an account which paid interest annually.
In the first year the account paid 2% compound interest.
In the second year the account paid 4% interest,
and in the third year the account paid 6% interest.
Work out the annual rate of interest (AER) of the account.

$$V = P\left(1 + \frac{2}{100}\right)\left(1 + \frac{4}{100}\right)\left(1 + \frac{6}{100}\right) = 1.124\,448\,P$$

Use the compound interest formula for 3 successive years with the correct value of r each time

Use the AER formula with $n = 3$ and $V = 1.124\,448P$

$$\text{AER} = 100 \times \left(\left(\frac{1.124448\,P}{P}\right)^{\frac{1}{3}} - 1\right) = 100 \times \left(1.124448^{\frac{1}{3}} - 1\right) = 3.99\%$$

NB As a way of checking, Adam's investment should give the same return as if he had invested in an account paying 3.99% p.a. compound interest for 3 years

$$P \times \left(1 + \frac{3.99}{100}\right)^3 = 1.1245\ldots \times P$$

which compares well with the $1.124448P$ above, the difference being due to the rounding of the AER to 2 decimal places.

Example 5

Holly invests £10000 in an account with an annual equivalent rate AER of 5%. She gets the interest paid half yearly. Work out the value of her first half yearly interest payment.
Suppose her half yearly interest payment rate is x%, then:

$$\left(1 + \frac{x}{100}\right)^2 = \left(1 + \frac{5}{100}\right)$$

This line comes from using compound interest for two successive half years and setting it equal to using 5% for 1 year. This line will be true no matter how much money is invested.

$$\text{So: } 1 + \frac{x}{100} = \sqrt{1.05} \quad x = 100 \times (\sqrt{1.05} - 1) = 2.469\,5\,\%$$

The amount of money added to the account is £10 000 × 2.469 5 % = £246.95.

Exercise 6B

B

1 Work out the value of these investments in accounts paying annual compound interest after the number of years stated.

	Initial Investment	Annual Interest rate	Number of years
a	£5000	5%	3
b	£2000	4%	5
c	£500	3.5%	6
d	£250	2.8%	10
e	£750	4.7%	18

2 Bill invests £5000 in an account paying 4% compound interest p.a. for 6 years. Work out the total interest that the account earns. A03

3 Mr Smith invests £10 000 in a savings scheme for 6 years. The AER of the savings scheme is 3.2%. A03
Mr Smith will have to pay tax at 40% on the total interest he gets at the end of the 6 years.
Work out how much tax Mr Smith will have to pay on the investment.

4 Every year Jim invests £1000 in an account paying 3% compound interest p. a. Work out the amount of money in the account at the end of the third year. A02

5 Mrs Newton wants to invest some money to pay for her son to attend university. She plans to invest in an account which pays 4.8% per annum compound interest. How much will she have to invest so that the account is worth £6000 after 5 years? A03

6 Ravi has £8000 to invest. He intends to leave it in his account for 6 years. What rate, per annum, of compound interest will enable the value of the account to reach £10 000 after 6 years? A03

7 An account pays 6% compound interest per annum. How many years will the investment have to be in place before its value doubles? A03

8 Work out the annual equivalent rate (AER) for each of these investments. A03

	Initial Investment	Number of years (n)	Value of the investment after n years
a	£5000	3	£6000
b	£2000	5	£2200
c	£500	6	£720
d	£250	10	£318
e	£750	18	£1710.50

A

9 James invests £1000 in an account. For the first year the account paid interest at 5% p.a. For the second year the account paid interest at 3.5% p.a. Work out the annual equivalent rate (AER) of interest on this account. Give your answer correct to 4 significant figures. A02

10 Annette invested £2500 in an account. In the first year the interest rate was 3%, in the second year 5% and in the third year 7%. A03

a Work out the value of Annette's account at the end of 3 years.

b Work out the annual equivalent rate (AER). Give your answer correct to 4 significant figures.

A

AO3 **11** Naseem invests £20 000 in an account. For the first two years the account pays 4% per annum compound interest, and for the next three years the account pays 6% per annum compound interest.

- **a** Work out the value of the account after 5 years.
- **b** Work out the annual equivalent rate (AER). Give your answer correct to 4 significant figures.

AO2 **12** A savings plan lasts for 5 years. For the first year the interest rate is 2%. The interest rate increases by 1% every year for the life of the savings plan.
Work out the annual equivalent rate (AER). Give your answer correct to 4 significant figures.

AO3 **13** An account pays 4% compound interest on the amount in the account every six months. What is the annual equivalent rate of interest?

A6.2 Cost of living index

Before you start

- You should be able to calculate with money.
- You should be able to calculate a percentage of an amount.

Why do this?

Basic money calculations are essential in modern life and having an understanding of the cost of living is useful when judging the value of wage rises.

Objectives

- Gain an understanding of financial mathematics.
- Be able to calculate wage increases which are in line with cost of living increases.

Get Ready

1. Work out 3% of £180.
2. Increase £320 by 5%.

Key Points

- The cost of living index is a measure of how prices increase. It is linked with the idea of inflation of prices.
- The cost of living index has a base year when the index is set equal to 100.
- The cost of living index increases by an amount each year, which depends on the costs of a typical set of items that people buy.

Example 1 Jim pays rent on a flat. Each year the rent increases in line with the cost of living index.

In 2010 the rent was £420 per month and the cost of living index was 100
In 2011 the cost of living index was 103.5
Work out what Jim's rent will be in 2011.

The cost of living increases by 3.5%. ← *The increase in the cost of living is 103.5 − 100 out of 100.*
Jim's rent will increase by 3.5% $= 420 \times \frac{3.5}{100} = 14.7$.
Jim's new rent will be £434.70 per month.

Example 2 Here is Mr Lincoln's bank statement from 1 April to 28 April. Some items are missing.

A deposit in an account happens when money is added to the account.

Date	Deposit (£)	Withdrawal (£)	Balance (£)
1.4.2012			3420.26
6.4.2012		200.00	3220.26
13.4.2012	312.51		3532.77
20.4.2012		250.00	....
28.4.2012	1250.00		....

The balance is the amount of money in the account.

a Write down how much was in the account on 1 April.
b Copy and complete Mr Lincoln's bank account.
c Mr Lincoln wants to know whether he can afford to pay a deposit of £4500 on a car. Can he afford it?

a £3420.26
b Missing items are £3282.77 and £4532.77
c Yes as the balance of his account is more than £4500.

Exercise 6C

1 John earns £250. He gets a wage rise of 10%. Work out his new wage.

2 Ben can buy 4 tins of tomatoes at 59p each or he can buy a bargain pack of 4 tins of tomatoes for £1.99. Work out how much he can save.

3 A litre of fuel costs 121.9p.
a Lizzie buys 25 litres of fuel. How much will she have to pay?
b Amir buys £40 worth of fuel. How much fuel does he buy?

4 Annie's rent is £112 per week. She gets a 10% reduction. Work out her new rent.

5 A student railcard costs £26. The railcard allows a student to buy rail tickets with $\frac{1}{3}$ off the normal price. Anya wants to get a rail ticket. The normal price is £114.
How much money can she save by buying a railcard and using it to reduce the price of the rail ticket?

F
A03

E
A03

E

AO2

6 **a** Lethna has £1.80. She wants to buy a drink and fries.
What are the different combinations that can she buy?

b Ken buys:
2 double burgers with cheese, 1 large portion fries and 1 large cola.
He pays with a £10 note. He gets the best price.
What change should he get?

Ben's Burger Bar

Burgers

Single burger	£0.85
Single burger with cheese	£0.95
Double burger	£1.55
Double burger with cheese	£1.70

Fries		**Cola**	
Regular	£0.65	Regular	£0.85
Large	£0.99	Large	£1.10

Meal Deals

Regular	
Single burger with cheese, regular fries and regular cola	£2.09
Large	
Double burger with cheese, large fries and large cola	£3.49

AO3

7

Natasha wants to buy 6 paper towel rolls. Work out how much she can save by using the special offer.

8 Javier gets the bus to and from work each day. He can get a daily return costing £2.90 or he can get a 5-day return costing £12.
How much will he save each week by buying a 5-day return?

9 Fred can buy a season ticket to watch his football team's home games. It will cost him £720 and allows him to attend all his team's home games.
Without a season ticket it will cost Fred £32 to attend each home game.
Fred's football team plays 23 home games.
Work out how much Fred would save by buying a season ticket.

D

10 Saeed earns £18 000 in a year. He does not pay tax on the first £6000 of the £18 000.
He pays tax 20% on the remainder.
Work out how much tax Saeed has to pay.

AO3

11 In 2009 Jenna found she had spent £3000 on rent, £800 on heating and £400 on rates.
In 2010, her rent for the year increased by 5%, heating by 15% and rates by 10%.
Work out the total increase in the amount of money that Jenna spent on these three items in 2010.

12 Oscar buys a car. The cash price of the car is £25 000.
Oscar pays a deposit of 30% of the cash price, followed by 24 monthly payments of £800 each.
How much altogether does Oscar pay for the car?

13 On average the cost of living is 5% higher in Cambridge than in Swindon.
Sophie spends £25 000 each year living in Swindon. How much would it cost her to live in Cambridge?

* **14** Jodie buys a car. The cash price of the car is £24 000. A02
Jodie pays a deposit of 25%, followed by 24 monthly payments of £900 each.
Bob says that overall Jodie will be paying more than 120% of the cash price of the car.
Is Bob correct? Explain your answer.

15 The cost of living index was 100 in 2005. It increased by 3% by the start of 2006. A03
Leonie gets a pay rise at the start of 2006 in line with inflation.
In 2005 she earned £1400 per month.
How much would she earn each month after her pay rise?

16 The cost of living index was 100 in 2005. It increased to 110.8 in 2009.
The living costs of Steve's family increased in line with inflation. In 2005 that cost was £800 per week.
How much was it in 2009?

* **17** The cost of living index was 100 in 2005. It increased to 108.5 in 2008. A02
The cost of a litre of petrol in 2005 was 88p. The cost of a litre of petrol in 2008 was £1.03.
Did the cost of petrol go up by a bigger percentage than the cost of living?
Explain your answer.

18 The cost of living index was 100 in 2005. It increased to 114 in 2010. The national minimum adult wage in 2005 was £5.05 per hour. A03

a What would the national minimum adult wage have to be in 2010 to keep pace with inflation?
The national minimum wage for 16-17 year olds in 2005 was £3.00 per hour. In 2010 it was £3.57.

***b** Has the national minimum wage for 16-17 year olds kept pace with inflation?
Explain your answer.

* **19** In 2007, Fran earned £20 000 per year. She spent 15% of her earnings on rent. A02 A03
By 2009, Fran's wage had increased by 5%. Her rent was now £3500 per year.
Does Fran spend a greater or smaller percentage of her earnings on rent in 2009 than she did in 2007?
You must give a reason for your answer.

20 Rail operators are allowed to raise fares by the cost of living index increase + 1%. In 2010, the cost of living increase was 4.5%.
The fare from Bristol to London in 2010 was £120.
What is the new fare in 2011 assuming the rail operator applies the maximum increase?

Review

- When £P is invested in an account paying r% compound interest per annum (p. a.), the value, £V, of investment after n years is given by:
$V = P\left(1 + \frac{r}{100}\right)^n$
- When £P is invested in an account for n years to produce an investment of value £V, the annual equivalent rate of interest (AER) is given by:
$\alpha = 100\left(\left(\frac{V}{P}\right)^{\frac{1}{n}} - 1\right)$ where: $\left(\frac{V}{P}\right)^{\frac{1}{n}} = \sqrt[n]{\left(\frac{V}{P}\right)}$
- The cost of living index gives information about the increase in cost of a set of typical items for a family over one year.

Answers

Chapter 6

A6.1 Get Ready answers

1 £6240
2 a 1024 b 7299.917
3 3

Exercise 6A

1 £2121.80
2 £2015.87

Exercise 6B

1 a £5788.13 b £2433.31 c £614.63
d £329.51 e £1714.36
2 $5000 \times 1.04^6 = £\,6326.60$ Interest = £1326.60
3 $10\,000 \times 1.032^6 - 10\,000 = £2080.31$
Tax = £832.12
4 $1000 \times 1.03^3 + 1000 \times 1.03^2 + 1000 \times 1.03 = £3183.63$
5 $P \times \left(1 + \frac{4.8}{100}\right)^5 = 6000 \quad P = \frac{6000}{1.048^5} = £4746.19$
6 $r = 100 \times \left(\left(\frac{10000}{8000}\right)^{\frac{1}{6}} - 1\right) = 3.79\%$
7 $P \times 1.06^n = P \times 2$
T&I gives $n = 11.9$ so after 12 full years.
8 a $100 \times \left(\left(\frac{6000}{5000}\right)^{\frac{1}{3}} - 1\right) = 6.27\%$
b $100 \times \left(\left(\frac{2200}{2000}\right)^{\frac{1}{5}} - 1\right) = 1.92\%$
c $100 \times \left(1.44^{\frac{1}{6}} - 1\right) = 6.27\%$
d $100 \times \left(1.272^{\frac{1}{10}} - 1\right) = 2.44$
e $100 \times \left(\left(\frac{1710.50}{750}\right)^{\frac{1}{18}} - 1\right) = 4.69\%$
9 $V = 1000 \times 1.05 \times 1.035 = £1086.75$
$\alpha = 100 \times \left(\left(\frac{1086.75}{1000}\right)^{\frac{1}{2}} - 1\right) = 4.247\%$
10 a $2500 \times 1.03 \times 1.05 \times 1.07 = £2893.01$
b $100 \times \left(\left(\frac{2893.10}{2500}\right)^{\frac{1}{3}} - 1\right) = 4.988\%$
11 a $20\,000 \times 1.04^2 \times 1.06^3 = £25\,764.06$
b $100 \times \left(\left(\frac{25\,764.06}{20\,000}\right)^{\frac{1}{5}} - 1\right) = 5.195\%$
12 $V = P \times 1.02 \times 1.03 \times 1.04 \times 1.05 \times 1.06$
$\alpha = 100 \times \left(\left(\frac{1.02 \times 1.03 \times 1.04 \times 1.05 \times 1.06}{1}\right)^{\frac{1}{5}} - 1\right)$
$= 3.990\%$
13 $\alpha = 100 \times (1.04^2 - 1) = 8.16\%$

A6.2 Get Ready answers

1 £5.40
2 £336

Exercise 6C

1 £275
2 236p − 199p = 37p
3 a £30.47 or £30.48 b 32.81 litres
4 £100.80
5 $\frac{114}{3} = 38$ so £12
6 a Regular fries with regular cola, Regular fries with large cola
b £4.81
7 £1.78
8 £2.50
9 £16
10 £2400
11 £150 + £120 + £40 = £310
12 £7500 + £19 200 = £26 700
13 £26 250
14 No. £6000 + £21 600 = £27 600, 1.20 × £24 000 = £28 800
15 £1442
16 £886.40 per week
17 88p × 1.085 = 95.48p which is less than 103p so the price of petrol has risen faster
18 a £5.76
b 300p × 1.14 = 342p so is above inflation.
19 New wage = £21 000.
New percentage = $\frac{3500}{21\,000} \times 100 = 16.7\%$, which is greater than 2007.
20 £126.60