**Pearson/Edexcel**

**GCSE (9-1) Mathematics (1MA1)**

**Problem-solving questions 4**

**Higher Tier**

**Time: 1 hour 30 minutes**

You should have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Calculator not permitted in questions with \*

**1.** The diagram shows an edge of a cliff, *AD*, with a zip line, *AB*.

*A*

*B*

*C*

*D*

*E*

 29 m

2°

 14 m

The time taken for Tom to travel down the zip line, *AB*, is 19 seconds.

Work out Tom’s average speed, in m/s, to travel down the zip line, *AB*.

Give your answer correct to 3 significant figures.

**(Total for question 1 is 3 marks)**

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**2\*.** *ABC* is a triangle with vertices *A*(1, −4), *B*(−5, 2) and *C*(11,6).

 *M* is the midpoint of *BC*.

 The line *L* passes through M and is perpendicular to *AC*.

 Show that the line *L* passes through the midpoint of *AC*.

**(Total for question 2 is 4 marks)**

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**3\*.** f and g are functions such that

 f(*x*) = 2*x* – 3 g(*x*) = *x*2 – 4

(a) Given that h(*x*) = gf(*x*), express h(*x*) in the form *p*(*x* + *q*)2 + *r*.

**(4)**

(b) State the coordinates of the turning point of the graph with equation h(*x*).

**(1)**

**(Total for question 3 is 5 marks)**

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**4.** Mary sells sausage rolls in her shop.

On Saturday she sold 20% more sausage rolls than on Friday.

On Sunday she sold 25% fewer sausage rolls than on Saturday.

On Sunday she sold 81 sausage rolls.

Work out the total number of sausage rolls sold on Friday, Saturday and Sunday.

**(Total for question 4 is 4 marks)**

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**5\*.** The diagram shows the graph of *y* = −4*x* + *c*

*y*

*x*

*c*

× (2, 10)

× (4, *a*)

Work out the value of *a* and *c*.

You must show your working.

**(Total for question 5 is 4 marks)**

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**6\*.** A box contains apples.

There are three times as many red apples as green apples in the box.

  of the red apples are rotten.

  of the green apples are rotten.

The number of rotten apples in the box is 35

Work out the total number of apples in the box.

**(Total for question 6 is 4 marks)**

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**7.**

*A*

*B*

*C*

*D*

*E*

10.4 m

65°

9.2 m

*ABE* and *DCE* are congruent triangles.

*BEC* is a sector of a circle with centre *E*.

*AB* = 10.4 m

*AE* = 9.2 m

Angle *BEC* = 65°

The area of sector *BEC* = 80 m2.

Work out the total area of the two triangles.

**(Total for question 7 is 5 marks)**

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**8.** The diagram shows a rectangular label taken off a cylindrical tin.

11 cm

24 cm

44 cm

The label covers the curved surface of the tin with no overlap.

Work out the volume of the cylindrical tin.

Give your answer correct to 3 significant figures.

**(Total for question 8 is 4 marks)**

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**9\*.** There are 315 counters in a box.

There are only red counters, green counters and blue counters in the box.

The ratio of the number of red counters to the number of green counters is 4 : 3

The ratio of the number of green counters to the number of blue counters is 2 : 7

Work out the difference between the number of red counters and the number of blue counters.

**(Total for question 9 is 4 marks)**

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**10\*.** Here are two sheets of cards.

 Card *X* (labelled *ABCD*) and card *Y* (labelled *EFGH*)are parallelograms.

*EE*

*F*

*G*

*H*

(2*x* − 1) m

**Card *Y***

*E*

*F*

*G*

*H*

**Card *X***

*A*

*B*

*C*

*D*

(*x* + 1) m

Card *X* has height (*x* + 1) m and area 4 m2.

Card *Y* has height (2*x* – 1) m and area 6 m2.

The sum of length *AD* and length *EH* is 8 m.

Work out the length of *AD* and the length of *EH*.

You must show your working.

**(Total for question 10 is 6 marks)**

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**11.** *ABD* and *BDC* are triangles.

*A*

*B*

*C*

*D*

38°

56°

47°

24 cm

17 cm

Work out the area of triangle *ABD*.

Give your answer correct to 3 significant figures.

**(Total for question 11 is 5 marks)**

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**12.** . The speed-time graph shows the speed of a car during the first 90 seconds of a journey.

Time (seconds)

0

20

40

60

80

100

Speed (m/s)

The car accelerates, then travels at a steady speed of *v* m/s and then decelerates.

The total distance travelled by the car in the first 90 seconds is 2.08 km.

Work out the value of *v* in m/s.

**(Total for question 12 is 4 marks)**

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**13\*.** Here is a number grid with a shaded T-shape.

The T-shape can be translated in any direction on the grid.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |

Prove algebraically that the sum of the numbers in any of these translated T-shapes is a multiple of 5

**(Total for question 13 is 3 marks)**

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**14\*.** The diagram shows a sheet of metal.

*A*

*B*

*C*

*D*

*E*

*F*

*G*

*H*

*I*

*J*

3 cm

(*x* + 1) cm

*ABCJ*, *GHIJ* and *CDEF* are squares.

*JCFG* is a rectangle.

*GH* = *FE*

*ED* = 3 cm

*AB* = (*x* + 1) cm

The total area of the sheet of metal is 88 cm2

Work out the value of *x*.

You must show your working.

**(Total for question 14 is 5 marks)**

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**15\*.** Here is a rectangle *ABCD* and a triangle *EFG*.

*E*

*F*

*G*

$\left(\sqrt{3}-1\right)$ cm

*h* cm

*A*

*B*

*C*

*DA*

$\left(3+\sqrt{5}\right)$ cm

$\left(3-\sqrt{5}\right)$ cm

The area of triangle *EFG* is twice the area of rectangle *ABCD*.

Work out the height, *h* cm, of triangle *DEF*.

Give your answer in the form $a(\sqrt{3}+b)$ where *a* and *b* are integers.

**(Total for question 15 is 4 marks)**

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| **Qn** | **Answer** | **Mark** | **Notes** |
| --- | --- | --- | --- |
| **1** | 22.6 | 3 | P1 Process to work out the length of *AB*, e.g. *AB* = (29 – 14) ÷ sin 2 (= 429.8056….)P1 Process to work out Tom's speed, e.g. “429.8056” ÷ 19 A1 22.6(213...)  |
| **2\*** | A correct comment with working | 4 | P1 Process to find the midpoint *M* or the midpoint of *AC*, e.g. or (3,4) OR or (6,1)P1 Process to find the gradient of *AC*, e.g.  (= 1)P1 Process to find the gradient of line joining *M* to the midpoint of *AC*, e.g.  (= −1)C1 A correct comment eg since gradient of *AC* × gradient of line joining *M* to the midpoint of *AC* = −1, the lines are perpendicular so *L* passes through the midpoint of *AC* |
| **3\*(a)** | 4(*x* –  )2 – 4 | 4 | P1 Process to find gf(*x*), e.g. (2*x* – 3)2 – 4 P1 Process to simplify gf(*x*) into the form *ax*2 + *bx* + *c*, e.g. 4*x*2 − 12*x* + 9 − 4 (= 4*x*2 − 12*x* + 5)P1 Process to express gf(*x*) in the form *p*(*x* + *q*)2 + *r*, e.g. 4[(*x* –  )2 – 9/4] + 5 e.g. 4(*x* – )2 – 9 + 5 A1 4(*x* – )2 – 4 oe |
| **(b)** | ( , −4) | 1 | B1 (“” , “−4”) |
| **4** | 279 | 4 | P1 Process to find the number of sausage rolls sold on Saturday, e.g. 1.2 × *x* (= 1.2*x*)P1 Process to find the number of sausage rolls sold on Sunday, e.g. 0.75 × “1.2*x*” (= 0.9*x*)P1 Process to set up an equation for the number sausage rolls sold on Sunday, e.g. 0.9*x* = 81 or *x* = 81 ÷ 0.9 (= 90)A1 279 |
| **5\*** | *c* = 18*a* = 2 | 4 | P1 Process to work out the value of *c*, e.g. 10 = −4(2) + *c* P1 Process to work out the value of *a*, e.g. *a* = −4(4) + “*c*” A1 for *c* = 18A1 for *a* = 2 |
| **6\*** | 200 | 4 | P1 Process to find the proportion of rotten red apples or rotten green apples of the total number of apples, e.g. or P1 Process to set up the equation of the proportion of rotten red apples and rotten green apples, e.g. + = 35P1 Process to solve the equation, e.g. 20*x* + 8*x* = 35 × 8 × 20A1  |
|  |  |  | OR  |
|  |  |  | P1 Process to set up an equation for total of rotten apples, e.g. P1 Process to eliminate one variable, e.g. *R* = 3*G* and P1 Process to find *G* and *R*, e.g. *G* = 50 and *R* = 3(50)A1 |
| **7**  | 92.(1...) | 5 | P1 Process to start to find the radius of the sector substitute correct values into equation of area of sector of circle, e.g. *π* × *r*2 × = 80P1 Full process to find the radius, e.g. *r* =  (= 11.8758) P1 Process to find an angle in triangle *ABE* or triangle *DCE*, e.g. A = cos-1 = (74.305°)P1 Process to work out the area of triangle *ABE* or triangle *DCE*, e.g. 0.5 × 10.4 × 9.2 × sin “74.305” (= 46.056)A1 92.(1...)  |
| **8** | 504 cm3 | 4 | P1 Process to find the radius or diameter of the tin, e.g. 2π*r* = 24 or πD = 24P1 Process to find the radius of the tin, e.g. r = 24 ÷2π (= 3.8197……..) or *r* = D ÷ 2 = (24 ÷ π) ÷ 2P1 Process to find the volume of the tin, e.g. V= π × (“3.8197…”)2 × 11A1 504 cm3 (must have units) |
| **9\*** | 117 | 4 | P1 Process to combine the ratios by using a common multiple, e.g. (4 : 3) × 2 and (2 : 7) × 3P1 A complete process to combine the ratios, e.g. 8 : 6 : 21P1 Process to find the difference between the number of red counters and blue counters, e.g.  × 315 (= 72) and  × 315 (= 189)or  × 315 (= 117)A1 for 117 |
| **10\*** | *AD* = 2 and *EH* = 6 | 6 | P1 Process to find length *AD* or *EH* ,e.g.  or P1 Process to set up an equation, e.g.  +  = 8P1 Process to reduce the equation to a*x*² + b*x* + c = 0 e.g. 8*x*² − 3*x* − 5 = 0 oe P1 Process to solve quadratic equation, e.g. (8*x* + 5)(*x* − 1) = 0P1 Process to find length *AD* or *EH*, e.g.  or A1 *AD* = 2 and *EH* = 6 (clearly identified) |
| **11** | 69.1 | 5 | P1 Process to find the length *BD*, e.g. 242 + 172 – 2(24 × 17 × cos38) (= 221.983)P1 A complete process to find the length *BD*,e.g.  (= 14.899)P1 Process to find the length *AD* or the length *AB*, e.g. *AD* =  × sin 56 (= 12.67682)e.g. *AB* =  × sin 47 (= 11.18313)P1 Process to find the area of triangle *ABD*, e.g.  × “14.899” × “12.67682” × sin 47 (= 69.1)e.g.  × “14.899” × “11.18313” × sin 56 (= 69.1)A1 for 69.1 |
| **12** | 32 | 4 | P1 Process to work out the area of a section under the curve e.g. (60 − 20)*v* (= 40*v*) or (0.5)(20 − 0)*v* (= 10*v*) or(0.5)(90 −60)*v* (= 15*v*) P1 Process to work out the total area under the curve, e.g. "40*v*" + "10*v*" + "15*v*" (= 65*v*)P1 Process to find *v*, e.g. "65*v*"= 2.08 e.g. "65*v*"= 2080 A1 for 32 |
| **13\*** | 5(*n* + 7) | 3 | P1 Process to set up an algebraic expression for the T-shape, e.g. *n* + *n* + 1 + *n* + 2 + *n* + 11 + *n* + 21 P1 Process to simplify the algebraic expression, e.g. 5*n* + 35C1 for 5(*n* + 7) or equivalent explanation  |
| **14\*** | 6 | 5 | P1 Process to work out the area of *JCFG* or *ABCJ* or (*ABCJ* and *CDEF*), e.g. (*x* + 1)(*x* + 1) (= *x*2 + 2*x* +1) or 3(*x* + 1) (= 3*x* + 3) or 2 × (3× 3) (= 18)P1 Process to set up an equation for the area of the shape, e.g. “*x*2 + 2*x* +1” + “3*x* + 3” + “18” = 88 P1 Process to simplify the quadratic equation into the form *ax*2 + *bx* + c = 0 e.g. “*x*2 + 5*x* – 66 = 0”P1 Process to solve the quadratic equation, e.g. (*x* + 11)(*x* – 6) = 0A1 for *x* = 6 |
| **15\*** | $$8\left(\sqrt{3}+1\right)$$ | 4 | P1 Process to set up an equation, e.g. *h* = 2 × “4”P1 Process to rearrange for *h*, e.g. *h* = P1 Process to rationalise the denominator, e.g. *h* =A1 for (Accept *a* = 8 and *b* = 1)  |